Date: 12/10/2021

**Q1.** Evaluate: $\lim\limits_{x \to 1} \dfrac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$.

**Q2.** Evaluate: $\lim\limits_{x \to 2} \dfrac{x^2 - 5x + 6}{(x^2 - 4)}$.

**Q3.** Evaluate: $\lim\limits_{x \to 3} \dfrac{x^4 - 81}{x^2 - 9}$

**Q4.** Evaluate: $\lim\limits_{x \to -\frac{1}{2}} \dfrac{8x^3 + 1}{2x + 1}$

**Q5.** Evaluate: $\lim\limits_{x \to \frac{1}{2}} \dfrac{4x^2 - 1}{2x - 1}$

**Q6.** If $f(x) = \begin{cases} x & x < 0 \\ 1 & x = 0 \\ x^2 & x > 0 \end{cases}$ Find $\lim\limits_{x \to 0} f(x)$ if it exists.

**Q7.** If $f(x) = \begin{cases} \cos x & x \geq 0 \\ x + k & x < 0 \end{cases}$. Find the value of $k$, such that $\lim\limits_{x \to 0} f(x)$ exists.

**Q8.** If $f(x) = \begin{cases} 1 + x, x > 0 \\ x, x < 0 \end{cases}$, then find the limit of $f(x)$ when $x \to 0$.

**Q9.** Find the limit: $\lim\limits_{x \to -1} [1 + x + x^2 + ... + x^{10}]$.

**Q10.** Find the limit: $\lim\limits_{x \to 3} [x(x + 1)]$.

**Q11.** Find the limit: $\lim\limits_{x \to 1} [x^3 - x^2 + 1]$.

**Q12.** Find the limit:

$$\lim_{x \to 1} \left[ \frac{x^2 + 1}{x + 100} \right].$$

**Q13.** Find the limit:

$$\lim_{x \to 2} \left[ \frac{x^2 - 4}{x^3 - 4x^2 + 4x} \right].$$

**Q14.** Find the limit:

$$\lim_{x \to 2} \left[ \frac{x^3 - 4x^2 + 4x}{x^2 - 4} \right].$$

**Q15.** Evaluate the limit: $\lim\limits_{x \to \pi} \left( x - \dfrac{22}{7} \right)$.

**Q16.** Find the limit:

$$\lim_{x \to 1}\left[\frac{x-2}{x^2-x}-\frac{1}{x^3-3x^2+2x}\right].$$

**Q17.** Find the limit:

$$\lim_{x \to 2}\left[\frac{x^3-2x^2}{x^2-5x+6}\right].$$

**Q18.** Evaluate the limit: $\lim_{r \to 1} \pi r^2$.

**Q19.** Evaluate the limit:

$$\lim_{x \to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}.$$

**Q20.** Evaluate the limit: $\lim_{z \to 1}\frac{z^{1/3}-1}{z^{1/6}-1}$.

**Q21.** Evaluate the limit: $\lim_{x \to 0}\frac{ax+b}{cx+1}$.

**Q22.** Evaluate the limit: $\lim_{x \to -1}\frac{x^{10}+x^5+1}{x-1}$.

**Q23.** Evaluate the limit: $\lim_{x \to 4}\frac{4x+3}{x-2}$.

**Q24.** Find $\lim_{x \to 0} f(x)$, where

$$f(x)=\begin{cases}\frac{x}{|x|}, & x \neq 0\\ 0, & x=0\end{cases}.$$

**Q25.** Find $\lim_{x \to 1} f(x)$, where

$$f(x)=\begin{cases}2x-1, & x \le 1\\ -x^2-1, & x>1\end{cases}.$$

**Q26.** Let $a_1, a_2, ..., a_n$ be fixed real numbers and define a function:

$$f(x)=(x-a_1)(x-a_2)\ ...\ (x-a_n).$$

What is $\lim_{x \to a_1} f(x)$? For some $a \neq a_1, a_2, ..., a_n$, compute $\lim_{x \to a} f(x)$.

**Q27.** If the function $f(x)$ satisfies $\lim_{x \to 1}\frac{f(x)-2}{x^2-1}=\pi$, evaluate $\lim_{x \to 1} f(x)$.

**Q28.** Evaluate: $\lim_{x \to 5}\frac{x^2-4x+3}{x^2+2x-3}$.

**Q29.**

If $f(x)=\begin{cases}x-[x] & x<2\\ 4 & x=2\\ 3x-5 & x>2\end{cases}$. Find $\lim_{x \to 2} f(x)$.

**Q30.** For what values of $P$ does the limit $\lim_{x \to 1} f(x)$ exist where f is defined by the rule

$$f(x)=\begin{cases}2Px+3 & \text{if} \quad x<1\\ 1-Px^2 & \text{if} \quad x>1\end{cases}$$

**Q31.** Evaluate: $\lim\limits_{h \to 0} \dfrac{\log_e(1+2h) - 2\log_e(1+h)}{h^2}$.

**Q32.** If $f(x) = \begin{cases} \dfrac{x - |x|}{x} & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$. Show that $\lim\limits_{x \to 0} f(x)$ does not exist.

**Q33.** Let $f(x) = \begin{cases} a + bx & x < 1 \\ 4 & x = 1 \\ b - ax & x > 1 \end{cases}$ and if $\lim\limits_{x \to 1} f(x) = f(1)$. Find the values of $a$ and $b$.

**Q34.** Evaluate the right hand limit of the function

$$f(x) = \begin{cases} \dfrac{|x - 6|}{x - 6}, & x \neq 6 \\ 0, & x = 6 \end{cases} \quad \text{at} \quad x = 6.$$

**Q35.** Evaluate the left hand limit of the function

$$f(x) = \begin{cases} \dfrac{|x - 4|}{x - 4}, & x \neq 4 \\ 0, & x = 4 \end{cases} \quad \text{at} \quad x = 4.$$

**Q36.** Evaluate the left hand and right hand limits of the following functions at $x = 2$.

$$f(x) = \begin{cases} 2x + 3, & \text{if } x \leq 2 \\ x + 5, & \text{if } x = 2 \end{cases} \quad \text{at} \quad x = 2.$$

Does $\lim\limits_{x \to 2} f(x)$ exists.

**Q37.** Let $f(x)$ be a function defind by

$$f(x) = \begin{cases} 6x - 6, & \text{if } x \leq 3 \\ 2x + k, & \text{if } x > 3 \end{cases}$$ Find $k$, if $\lim\limits_{x \to 3} f(x)$ exists.

**Q38.** Find $\lim\limits_{x \to 0} f(x)$ and $\lim\limits_{x \to 1} f(x)$, where $f(x) = \begin{cases} 2x + 3, & x \leq 0 \\ 3(x + 1), & x > 0 \end{cases}$.

**Q39.** For what integers $m$ and $n$ does both $\lim\limits_{x \to 0} f(x)$ and $\lim\limits_{x \to 1} f(x)$ exist? If

$$f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^2 + m, & x > 1 \end{cases}$$

**Q40.** Find $\lim\limits_{x \to 5} f(x)$, where $f(x) = |x| - 5$.

**Q41.** Evaluate the limit: $\lim\limits_{x \to 3} \dfrac{x^4 - 81}{2x^2 - 5x - 3}$.

**Q42.** Evaluate the limit: $\lim\limits_{x \to 0} \dfrac{(x+1)^5 - 1}{x}$.

**Q43.** Evaluate the limit: $\lim\limits_{x \to 2} \dfrac{3x^2 - x - 10}{x^2 - 4}$.

**Q44.** Evaluate: $\lim\limits_{x \to 0} \dfrac{(1+x)^4 - 1}{x}$.

**Q45. Evaluate:** $\lim_{x \to a} \frac{(x+2)^{5/3} - (a+2)^{5/3}}{(x-a)}$.

Date: 12/10/2021

**S1.**

$$\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a} = \frac{a+b+c}{c+b+a} = 1.$$

**S2.**

$$\lim_{x\to 2}\frac{x^2-5x+6}{(x^2-4)} = \lim_{x\to 2}\frac{\cancel{(x-2)}\,(x-3)}{\cancel{(x-2)}\,(x+2)}$$

$$= \lim_{x\to 2}\frac{(x-3)}{(x+2)} = -\frac{1}{4}.$$

**S3.**

$$\lim_{x\to 3}\frac{x^4-81}{x^2-9} = \lim_{x\to 3}\frac{(x^2)^2-(9)^2}{(x^2-9)}$$

$$= \lim_{x\to 3}\frac{(x^2+9)\,\cancel{(x^2-9)}}{\cancel{(x^2-9)}} = 18.$$

**S4.**

$$\lim_{x\to -\frac{1}{2}}\frac{(2x)^3+1^3}{(2x+1)} = \lim_{x\to -\frac{1}{2}}\frac{\cancel{(2x+1)}\,(4x^2-2x+1)}{\cancel{(2x+1)}}$$

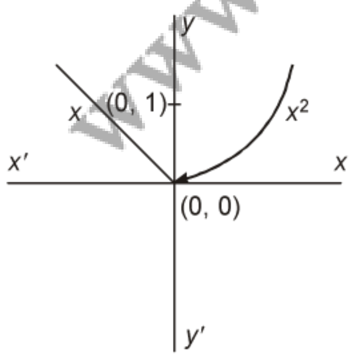
$$= \lim_{x\to -\frac{1}{2}}(4x^2-2x+1) = 3.$$

**S5.**

$$\lim_{x\to \frac{1}{2}}\frac{(2x)^2-1^2}{(2x-1)} = \lim_{x\to \frac{1}{2}}\frac{\cancel{(2x-1)}\,(2x+1)}{\cancel{(2x-1)}}$$

$$= \lim_{x\to \frac{1}{2}}(2x+1) = 2.$$

**S6.**



By graph clearly the limiting value at $x = 0$ is zero hence.

$$\lim_{x \to 0} f(x) = 0$$

**S7.** L.H.L. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \cos x = 1$

at $x = 0, \quad f(x) = 1$

R.H.L. $\lim_{x \to 0^+} f(x) = 0 + k = k$

$\because$ R.H.L. = L.H.L.

$\Rightarrow$ $k = 1$.

**S8.** $f(x) = -x$

$$f'(x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h} = \frac{x + h - x}{h} = 1$$ **Ans.**

[$\because$ $f(x) = 1 + x$ when $x > 0$]

$\therefore$ $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$ and so $\lim_{x \to 0} f(x)$ does not exist.

**S9.** Let, $\lim_{x \to -1} [1 + x + x^2 + \ldots + x^{10}] = 1 + (-1) + (-1)^2 + \ldots + (-1)^{10}$

$= 1 - 1 + 1 + \ldots + 1 = 1.$

**S10.** Let, $\lim_{x \to 3} [x(x+1)] = 3(3 + 1) = 3(4) = 12.$

**S11.** Let, $\lim_{x \to 1} [x^3 - x^2 + 1] = 1^3 - 1^2 + 1 = 1.$

**S12.** All the functions under consideration are rational function. Hence, we first evaluate these functions at the prescribed points. If this is of the form $\frac{0}{0}$, we try to rewrite the function cancelling the factors which are causing the limit to be of the form $\frac{0}{0}$.

We have, $\lim_{x \to 1} \frac{x^2 + 1}{x + 100} = \frac{1^2 + 1}{1 + 100} = \frac{2}{101}$.

**S13.** All the functions under consideration are rational function. Hence, we first evaluate these functions at the prescribed points. If this is of the form $\frac{0}{0}$, we try to rewrite the function cancelling the factors which are causing the limit to be of the form $\frac{0}{0}$.

Evaluating the function at 2, we get it of the form $\frac{0}{0}$.

Hence, $\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 4x^2 + 4x} = \lim_{x \to 2} \frac{(x+2)(x-2)}{x(x-2)^2}$

$$= \lim_{x \to 2} \frac{(x+2)}{x(x-2)} = \frac{2+2}{2(2-2)} = \frac{4}{0}.$$

which is not defined.

**S14.** All the functions under consideration are rational function. Hence, we first evaluate these functions at the prescribed points. If this is of the form $\frac{0}{0}$, we try to rewrite the function cancelling the factors which are causing the limit to be of the form $\frac{0}{0}$.

Evaluating the function at 2, it is of the form $\frac{0}{0}$.

Hence, $$\lim_{x \to 2} \frac{x^3 - 4x^2 + 4x}{x^2 - 4} = \lim_{x \to 2} \frac{x(x-2)^2}{(x+2)(x-2)}$$

$$= \lim_{x \to 2} \frac{x(x-2)}{(x+2)}$$

$$= \frac{2(2-2)}{2+2} = \frac{0}{4} = 0 \quad \text{as} \quad x \neq 2.$$

**S15.** $\because$ $$\lim_{x \to \pi} \left(x - \frac{22}{7}\right) = \lim_{x \to \pi} x - \lim_{x \to \pi} \frac{22}{7}$$

$$= \pi - \frac{22}{7}.$$

**S16.** All the functions under consideration are rational function. Hence, we first evaluate these functions at the prescribed points. If this is of the form $\frac{0}{0}$, we try to rewrite the function cancelling the factors which are causing the limit to be of the form $\frac{0}{0}$.

First, we rewrite the function as a rational function.

$$\left[\frac{x-2}{x^2 - x} - \frac{1}{x^3 - 3x^2 + 2x}\right] = \left[\frac{x-2}{x(x-1)} - \frac{1}{x(x^2 - 3x + 2)}\right]$$

$$= \left[\frac{x-2}{x(x-1)} - \frac{1}{x(x-1)(x-2)}\right]$$

$$= \left[\frac{x^2 - 4x + 4 - 1}{x(x-1)(x-2)}\right]$$

$$= \frac{x^2 - 4x + 3}{x(x-1)(x-2)}$$

Evaluating the function at $x = 1$, we get it of the form $\frac{0}{0}$.

Hence, $$\lim_{x \to 1}\left[\frac{x-2}{x^2 - x} - \frac{1}{x^3 - 3x^2 + 2x}\right] = \lim_{x \to 1} \frac{x^2 - 4x + 3}{x(x-1)(x-2)}$$

$$= \lim_{x \to 1} \frac{(x-3)(x-1)}{x(x-1)(x-2)}$$

$$= \lim_{x \to 1} \frac{x-3}{x(x-2)} = \frac{1-3}{1(1-2)} = 2.$$

We remark that we could cancel the term $(x - 1)$ in the above evaluation because $x \neq 1$.

**S17.** All the functions under consideration are rational function. Hence, we first evaluate these functions at the prescribed points. If this is of the form $\frac{0}{0}$, we try to rewrite the function cancelling the factors which are causing the limit to be of the form $\frac{0}{0}$.

Evaluating the function at 2, we get it of the form $\frac{0}{0}$.

Hence, $$\lim_{x \to 2} \frac{x^3 - 2x^2}{x^2 - 5x + 6} = \lim_{x \to 2} \frac{x^2(x-2)}{(x-2)(x-3)}$$

$$= \lim_{x \to 2} \frac{x^2}{(x-3)} = \frac{(2)^2}{2-3} = \frac{4}{-1} = -4.$$

**S18.** $\because$ $$\lim_{r \to 1} \pi r^2 = \pi \lim_{r \to 1} r^2 = \pi(1) = \pi.$$

**S19.** Evaluating the function at $-2$, we get it of the form $\frac{0}{0}$.

Hence, $$\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x+2} = \lim_{x \to -2} \frac{\frac{2+x}{2x}}{x+2}$$

$$= \lim_{x \to -2} \frac{2+x}{2x(x+2)}$$

$$= \lim_{x \to -2} \frac{1}{2x} = \frac{1}{2(-2)} = -\frac{1}{4}.$$

**S20.** Let $$\lim_{z \to 1} \frac{z^{1/3} - 1}{z^{1/6} - 1} = \lim_{z \to 1} \left[ \frac{z^{1/3} - 1}{z - 1} \div \frac{z^{1/6} - 1}{z - 1} \right]$$

$$= \lim_{z \to 1} \left[ \frac{z^{1/3} - 1}{z - 1} \right] \div \lim_{z \to 1} \left[ \frac{z^{1/6} - 1}{z - 1} \right]$$

$$= \frac{1}{3}(1)^{\frac{1}{3}-1} \div \frac{1}{6}(1)^{\frac{1}{6}-1} = \frac{1}{3}(1)^{\frac{-2}{3}} \div \frac{1}{6}(1)^{\frac{-5}{6}}$$

$$= \frac{1}{3} \div \frac{1}{6} = \frac{6}{3} = 2.$$

**S21.** $\because$ $$\lim_{x \to 0} \frac{ax + b}{cx + 1} = \frac{a(0) + b}{c(0) + 1} = b.$$

**S22.** $\because$
$$\lim_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1} = \frac{\lim_{x \to -1} (x^{10} + x^5 + 1)}{\lim_{x \to -1} (x - 1)} = \frac{1^{10} + 1^5 + 1}{-1 - 1}$$
$$= \frac{1 + 1 + 1}{-2} = \frac{-3}{2}.$$

**S23.** $\because$
$$\lim_{x \to 4} \frac{4x + 3}{x - 2} = \frac{\lim_{x \to 4} (4x + 3)}{\lim_{x \to 4} (x - 2)}$$
$$= \frac{4(4) + 3}{(4 - 2)} = \frac{19}{2}.$$

**S24.** We have $x \to 0$, *i.e*, $x \to 0^-$ and $x \to 0^+$

Now, $x \to 0^- \Rightarrow \lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = -1$

$x \to 0^+ \Rightarrow \lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = 1$

*i.e.*, $\lim_{x \to 0} f(x)$ does not exist.

**S25.** For $x > 1$

Right hand limit = $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (-x^2 - 1)$

$= -(1)^2 - 1 = -2$

For $x < 1$

Left hand limit = $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2 - 1)$

$= (1)^2 - 1 = 0$

As $\lim_{x \to 1^+} f(x) \neq \lim_{x \to 1^-} f(x)$

$\therefore \lim_{x \to 1} f(x)$ does not exist.

**S26.** Since $f(x)$ is a polynomial of degree $n$, the $\lim_{x \to a_1} f(x)$ could be found by putting $x = a_1$ in $f(x)$.

Thus, $\lim_{x \to a_1} f(x) = (a_1 - a_1)(a_1 - a_2) \dots (a_1 - a_n)$

$= 0$

Similarly, $\lim_{x \to a} f(x) = (a - a_1)(a - a_2) \dots (a - a_n)$.

**S27.** According to the question:
$$\lim_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi$$

or $\frac{f(1)-2}{0} = \pi$

or $f(1) - 2 = 0$

or $f(1) = 2$

Now, $\lim_{x \to 1} f(x) = f(1) = 2.$

**S28.** $\lim_{x \to 5} \frac{x^2 - 3x - x + 3}{x^2 + 3x - x - 3} = \lim_{x \to 5} \frac{x(x-3) - 1(x-3)}{x(x+3) - 1(x+3)}$

$= \lim_{x \to 5} \frac{(x-1)(x-3)}{(x+3)(x-1)}$

$= \lim_{x \to 5} \frac{(x-3)}{(x+3)}$

$= \lim_{x \to 5} \frac{5-3}{5+3} = \frac{2}{8} = \frac{1}{4}.$

**S29.** L.H.L. $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x - [x]$

$= \lim_{h \to 0} f(2-h) = \lim_{h \to 0} (2-h) - [(2-h)]$

$= \lim_{h \to 0} (2 - h - 1) = \lim_{h \to 0} (1 - h) = 1$

R.H.L. $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x - 5)$

$= \lim_{h \to 0} f(2+h) = \lim_{h \to 0} (3(2+h) - 5)$

$= \lim_{h \to 0} (6 + 3h - 5) = \lim_{h \to 0} (1 + 3h) = 1$

L.H.L. = R.H.L.

Hence limit exists at $x = 2$ and equal to 1.

**S30.** L.H.L. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2Px + 3$

$= 2P + 3$

R.H.L $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 1 - Px^2$

$= (1 - P)$

$\because$ R.H.L. = L.H.L.

$\Rightarrow$. $2P + 3 = 1 - P \Rightarrow 3P = -2$

$\Rightarrow \qquad P = -\frac{2}{3}.$

**S31.** Given, $\lim_{h \to 0} \frac{\log_e(1+2h) - 2\log_e(1+h)}{h^2}$

$$= \lim_{h \to 0} \frac{\left\{2h - \frac{(2h)^2}{2} + \frac{(2h)^3}{3} - \dots\dots\dots\right\} - 2\left\{h - \frac{h^2}{2} + \frac{h^3}{3} - \dots\dots\right\}}{h^2}$$

$$= \lim_{h \to 0} \frac{(-h^2 + 2h^3 - \dots\dots)}{h^2} = \lim_{h \to 0}(-1 + 2h - \dots\dots) = -1 + 0 = -1.$$

**Alternative Method:**

$$= \lim_{h \to 0} \frac{\log_e(1+2h) - 2\log_e(1+h)}{h^2} = \lim_{h \to 0} \frac{\ln\left[\frac{(1+h^2)}{1+2h}\right]}{h^2}$$

$$= \lim_{h \to 0} \frac{\ln\left[\frac{h^2}{1+2h}\right]}{h^2} = \lim_{h \to 0} \frac{\ln\left(1 + \frac{h^2}{1+2h}\right)}{\left(\frac{h^2}{1+2h}\right).(1+2h)}$$

$$= \lim_{h \to 0} \frac{\ln\left(1 + \frac{h^2}{1+2h}\right)}{\left(\frac{h^2}{1+2h}\right)} . \lim_{h \to 0} \frac{1}{(1+2h)}$$

$$= -1 \times \frac{1}{(1+0)} = -1.$$

$$\left[\because \lim_{t \to 0} \frac{\ln(1+t)}{t} = 1, \text{here as } h \to 0, \frac{h^2}{1+2h} \to 0\right]$$

**S32.** L.H.L. of $f(x)$ at $x = 0$

$$= \lim_{x \to 0^-} f(x)$$

$$= \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{(0-h) - |(0-h)|}{(0-h)}$$

Putting $(x = 0 - h)$

$$= \lim_{h \to 0} \frac{-h - |-h|}{(-h)} = \lim_{h \to 0} \frac{-h - h}{-h}$$

$$= \lim_{h \to 0} \frac{-2h}{-h} = 2.$$

R.H.L. of $f(x)$ at $x = 0$

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{(0+h) - |(0+h)|}{0+h} = 0$$

$\because$ L.H.L. $\neq$ R.H.L. Hence limit does not exist.

**S33.** $f(x) = \begin{cases} a + bx & x < 1 \\ 4 & x = 1 \\ b - ax & x > 1 \end{cases}$

$\therefore$ L.H.L. $\lim_{x \to 1^-} f(x) = \lim_{h \to 0} [a + b(1-h)]$

Put $(x = 1 - h)$

$$= \lim_{h \to 0} (a + b - bh) = a + b$$

R.H.L. $\lim_{x \to 1^+} f(x) = \lim_{h \to 0} [b - a(1+h)]$

$$= (b - a)$$

$\because$ $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1)$

$=$ $a + b = b - a = 4$

$=$ $a = 0, \quad b = 4.$

**S34.** We have, R.H.L. of $f(x)$ at $x = 6$

$$\lim_{x \to 6^+} f(x) = \lim_{h \to 0} f(6+h)$$

$$= \lim_{h \to 0} \frac{|6+h-6|}{6+h-6} = \lim_{h \to 0} \frac{|h|}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1.$$

**S35.** We have, L.H.L. of $f(x)$ at $x = 4$

$$\lim_{x \to 4^-} f(x) = \lim_{h \to 0} f(4-h)$$

$$= \lim_{h \to 0} \frac{|4-h-4|}{4-h-4} = \lim_{h \to 0} \frac{|-h|}{-h}$$

$$= \lim_{h \to 0} \frac{h}{-h} = \lim_{h \to 0} (-1) = -1.$$

**S36.** We have, L.H.L. = $\lim_{x \to 2^-} f(x)$

Put $x = (2 - h)$

$= \lim_{x \to 2^-} (2x + 3)$

$= \lim_{h \to 0} [2(2 - h) + 3]$

$= 4 + 3 = 7.$

R.H.L. = $\lim_{x \to 2^+} f(x)$

$= \lim_{x \to 2^+} (x + 5)$

$= \lim_{h \to 0} (2 + h + 5)$

Put $x = (2 + h)$

$= 2 + 5 = 7.$

$\because$ R.H.L. = L.H.L. at $x = 2$.

= $\lim_{x \to 2} f(x)$ exists and it is equal to 7.

**S37.** We have,

$$f(x) = \begin{cases} 6x - 6, & \text{if } x \le 3 \\ 2x + k, & \text{if } x > 3 \end{cases}$$

L.H.L. at $x = 3$

$= \lim_{x \to 3^-} f(x)$

$= \lim_{h \to 0} f(3 - h)$

Put $x = (3 - h)$

$= \lim_{h \to 0} 6(3 - h) - 6$

$= 18 - 6 = 12.$

and R.H.L. at $x = 3$

$= \lim_{x \to 3^+} f(x)$

$= \lim_{h \to 0} f(3 + h)$

$= \lim_{h \to 0} 2(3 + h) - k = 6 - k$

if $\lim_{x \to 3} f(x)$ exists, then

$$\text{R.H.L.} = \text{L.H.L.}$$

$\Rightarrow$ $$12 = 6 - k$$

$\Rightarrow$ $$k = -6.$$

**S38.** Limit as $x \to 0$.

We have, $$f(x) = \begin{cases} 2x + 3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$$

For $x > 0$

$$\text{Right hand limit} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 3(x+1)$$

$$= 3(0 + 1) = 3$$

For $x < 0$

$$\text{Left hand limit} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (2x+3)$$

$$= 2(0) + 3 = 3$$

As $$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$\lim_{x \to 0} f(x) = 3$$

Now, limit $x \to 1$

For $x > 1$

$$\text{Right hand limit} = \lim_{x \to 1^+} f(x)$$

$$= \lim_{x \to 1^+} 3(x+1) = 3(1 + 1) = 6$$

For $x < 1$

$$\text{Left hand limit} = \lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1^-} (2x+3) = 2(-1) + 3 = 1$$

As $$\lim_{x \to 1^+} f(x) \ne \lim_{x \to 1^-} f(x)$$

$\lim_{x \to 1} f(x)$ does not exist.

**S39.** For a limit to exist at a point, left hand limit must be equal to the right hand limit at that point.

Now, for $\lim_{x \to 0} f(x)$ to exist, we have

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h)$$

$$= m \cdot 0 + n = n$$

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h)$$

$$= n \cdot 0 + m = m$$

Hence, $n = m$.

Thus $\lim_{x \to 0} f(x)$ exists for all $m = n$.

For $\lim_{x \to 1} f(x)$ to exist, we have

$$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h)$$

$$= n(1 - 0) + m = m + n$$

$$\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h)$$

$$= n(1 + 0)^2 + m = m + n$$

As $m + n = m + n$

$\lim_{x \to 1} f(x)$ exists for all integral values of $m$ and $n$.

**S40.** $$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} (|x| - 5)$$

Putting $x = 5 - h$

$$= \lim_{h \to 0^-} (|5 - h| - 5) \qquad [h \to 0^- \text{ as } x \to 5^-]$$

$$= \lim_{h \to 0^-} (5 - h - 5)$$

$$= \lim_{h \to 0^-} (-h) = 0$$

and $$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} (|x| - 5)$$

Putting $x = 5 + h$

$$= \lim_{h \to 0^+} (|5 + h| - 5) \qquad [h \to 0^+ \text{ as } x \to 5^+]$$

$$= \lim_{h \to 0^+} (5 + h - 5)$$

$$= \lim_{h \to 0^+} (h) = 0$$

Hence, $\lim_{x \to 5} f(x) = 0$.

**S41.** Evaluating the function at 3, we get it of the form $\frac{0}{0}$.

Hence, $$\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3} = \lim_{x \to 3} \frac{(x^2)^2 - (3^2)^2}{2x^2 - 6x + x - 3}$$

$$= \lim_{x \to 3} \frac{(x^2+3^2)(x^2-3^2)}{2x(x-3)+1(x-3)}$$

$$= \lim_{x \to 3} \frac{(x^2+9)(x+3)(x-3)}{(x-3)(2x+1)}$$

$$= \lim_{x \to 3} \frac{(x^2+9)(x+3)}{(2x+1)}$$

$$= \frac{(3^2+9)(3+3)}{2(3)+1} = \frac{18 \times 6}{7} = \frac{108}{7}.$$

**S42.** Given, $\lim_{x \to 0} \frac{(x+1)^5-1}{x}$

Evaluating the function at 0, we get it of the form 0/0.

Then, we try to rewrite the function cancelling the factors which are causing the limit to be of the form $\frac{0}{0}$.

Hence, $$\lim_{x \to 0} \frac{(x+1)^5-1}{x} = \lim_{x \to 0} \frac{\left(1+5x+\frac{5(5-1)}{2!}x^2+\ldots+x^5\right)-1}{x}$$

$$= \lim_{x \to 0} \frac{(1+5x+10x^2+\ldots+x^5)-1}{x}$$

$$= \lim_{x \to 0} \frac{5x+10x^2+\ldots+x^5}{x}$$

$$= \lim_{x \to 0} \frac{x(5+10x+\ldots+x^4)}{x}$$

$$= \lim_{x \to 0} (5+10x+\ldots+x^4) = \lim_{x \to 0} 5 = 5.$$

**S43.** Evaluating the function at 2, we get it of the form $\frac{0}{0}$.

Hence, $$\lim_{x \to 2} \frac{3x^2-x-10}{x^2-4} = \lim_{x \to 2} \frac{3x^2-6x+5x-10}{(x+2)(x-2)}$$

$$= \lim_{x \to 2} \frac{3x(x-2)+5(x-2)}{(x+2)(x-2)}$$

$$= \lim_{x \to 2} \frac{(x-2)(3x+5)}{(x+2)(x-2)}$$

$$= \lim_{x \to 2} \frac{3x+5}{x+2} = \frac{3(2)+5}{2+2} = \frac{11}{4}.$$

**S44.** Given $$\lim_{x \to 0} \frac{(1+x)^4-1}{x} = \lim_{x \to 0} \frac{((1+x)^2-1)((1+x)^2+1)}{x}$$

$$= \lim_{x \to 0} \frac{(1 + x^2 + 2x - 1)(1 + x^2 + 2x + 1)}{x}$$

$$= \lim_{x \to 0} \frac{(x^2 + 2x)(2 + x^2 + 2x)}{x}$$

$$= \lim_{x \to 0} (x + 2)(x^2 + 2x + 2)$$

$$= 2 \times 2 = 4.$$

**S45.** Given, $\lim_{x \to a} \frac{(x + 2)^{5/3} - (a + 2)^{5/3}}{(x - a)}$

Let $x + 2 = X$ and $a + 2 = A$

$\therefore$ $x \to a \Rightarrow x + 2 \to a + 2$

$\Rightarrow$ $X \to A.$

$$= \lim_{x \to a} \frac{(x + 2)^{5/3} - (a + 2)^{5/3}}{(x - a)}$$

$$= \lim_{X \to A} \frac{X^{5/3} - A^{5/3}}{(X - A)} = \frac{5}{3} A^{(5/3 - 1)}$$

$$= \frac{5}{3} \cdot A^{2/3} = \frac{5}{3} \cdot (a + 2)^{2/3}.$$

Date: 12/10/2021

**Q1.** Evaluate: $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$.

**Q2.** If $\lim_{x \to 1} \frac{x^k - 5^k}{x - 5} = 500$, then find the value of $k$.

**Q3.** Find the value of $\lim_{x \to 9} \left\{\frac{x^{3/2} - 27}{x - 9}\right\}$.

**Q4.** Find the value of $\lim_{x \to a} \frac{x^{-1} - a^{-1}}{x - a}$.

**Q5.** If $a > 0$ and $\lim_{x \to a} \frac{a^x - x^a}{x^x - a^a} = 1$, then find the value of $a$.

**Q6.** If $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to (m)} \frac{x^3 - m^3}{x^5 - m^5}$, then find the value of $m^2$.

Date: 12/10/2021

**S1.** Put $y = 1 + x$, so that $y \to 1$ as $x \to 0$.

Then, $$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{y \to 1} \frac{\sqrt{y} - 1}{y - 1}$$

$$= \lim_{y \to 1} \frac{y^{\frac{1}{2}} - 1^{\frac{1}{2}}}{y - 1}$$

$$= \frac{1}{2}(1)^{\frac{1}{2}-1} = \frac{1}{2}.$$

**S2.** Given, $$\lim_{x \to 1} \frac{x^k - 5^k}{x - 5} = 500$$

$\Rightarrow$ $$k.5^{k-1} = 500 \qquad \left[\because \lim_{x \to a} \frac{x^k - a^n}{x - a} = na^{n-1}\right]$$

$\Rightarrow$ $$k \,.\, 5^{k-1} = 4 \times 5^3$$

$\Rightarrow$ $$k = 4.$$

**S3.** Given, $\lim_{x \to 9} \left\{\frac{x^{3/2} - 27}{x - 9}\right\}$ $\left[\frac{0}{0} \text{ form}\right]$

$$= \lim_{x \to 9} \frac{\frac{3}{2}x^{1/2}}{1} = \frac{3}{2} \times \sqrt{9} = \frac{9}{2}. \qquad \left[\because \lim_{x \to a}\left(\frac{x^n - a^n}{x - a}\right) = na^{n-1}\right]$$

**S4.** Given, $\lim_{x \to a} \frac{x^{-1} - a^{-1}}{x - a}$ $\left[\frac{0}{0} \text{ form}\right]$

$$= \lim_{x \to a} \left\{\frac{-\frac{1}{x^2}}{1}\right\} = -\frac{1}{a^2}.$$

Another Method: $$\lim_{x \to a} \frac{x^{-1} - a^{-1}}{x - a} = -1a^{-2} = \frac{-1}{a^2}. \qquad \left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

**S5.** Given, $\lim_{x \to a} \frac{a^x - x^a}{x^x - a^a}$ $\left[\frac{0}{0} \text{ form}\right]$

$$= \lim_{x \to a} \frac{a^x \log a - ax^{a-1}}{x^x(1+\log x)}$$

$$\left[\begin{array}{l} \therefore\ y = x^x \ \Rightarrow\ \log y = x \log x \Rightarrow \frac{1}{y}\frac{dy}{dx} = x, \frac{1}{x} + \log x.1 \\ \Rightarrow\ \frac{dy}{dx} = y(1+\log x) = x^x(1+\log x) \end{array}\right]$$

$$= \frac{a^a \log a - a.a^{a-1}}{a^a(1+\log a)} = \frac{a^a(\log a - 1)}{a^a(\log a + 1)} = \frac{\log a - 1}{\log a + 1}$$

Now, $\lim_{x \to a} \frac{a^x - x^a}{x^x - a^a} = -1$

$\Rightarrow$ $\frac{\log a - 1}{\log a + 1} = -1 \Rightarrow \log a - 1 = -1 a - 1 \Rightarrow 2 \log a = 0$

$\Rightarrow$ $\log a = 0 \ \Rightarrow\ a = e^0 = 1.$

**S6.** L.H.S. $= \lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to 1} \frac{(x-1)(x+1)(x^2+1)}{(x-1)}$

$= \lim_{x \to 1} (x+1)(x^2+1)$

$= (1 + 1) \times (1^2 + 1) = 2 \times 2 = 4.$

R.H.S. $= \lim_{x \to m} \frac{x^3 - m^3}{x^5 - m^5} = \frac{3}{5} \cdot m^{-2}$ $\left[\because\ \lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} \cdot a^{m-n}\right]$

$\Rightarrow$ $\frac{3}{5} \cdot \frac{1}{m^2} = 4 \ \Rightarrow\ \frac{1}{m^2} = \frac{20}{3} \ \Rightarrow\ m^2 = \frac{3}{20}.$

Date: 12/10/2021

**Q1.** Evaluate: $\lim\limits_{x \to \infty} \sqrt{x}(\sqrt{x+a} - \sqrt{x})$.

**Q2.** If $G(x) = -\sqrt{25 - x^2}$, then find $\lim\limits_{x \to 1} \frac{G(x) - G(1)}{x - 1}$.

**Q3.** Evaluate $\lim\limits_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$.

**Q4.** Evaluate: $\lim\limits_{x \to \infty} x[\sqrt{x^2 + 6} - x]$.

**Q5.** Evaluate: $\lim\limits_{x \to 0} \left\{\frac{\sqrt{a+x} - \sqrt{a-x}}{x}\right\}$.

**Q6.** Evaluate: $\lim\limits_{x \to a} \frac{(\sqrt{a+2x} - \sqrt{3x})}{(\sqrt{3a+x} - 2\sqrt{x})}$.

**Q7.** Evaluate: $\lim\limits_{x \to 2} \frac{(\sqrt{1+\sqrt{2+x}} - \sqrt{3})}{(x-2)}$.

**Q8.** Evaluate: $\lim\limits_{x \to 2} \frac{\sqrt{x-2} + \sqrt{x} - \sqrt{2}}{\sqrt{x^2 - 4}}$.

**Q9.** Evaluate: $\lim\limits_{x \to 2a} \frac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$.

**Q10.** Evaluate: $\lim\limits_{x \to 2} \frac{\sqrt{x^2 + x - 3} - \sqrt{x+1}}{x - 2}$.

**Q11.** Evaluate: $\lim\limits_{x \to 0} \frac{\sqrt{x^2+1} - 1}{\sqrt{x^2+9} - 3}$.

**Q12.** Evaluate: $\lim\limits_{x \to \infty} (\sqrt{x^2 + 2x - 1} - x)$.

**Q13.** Evaluate: $\lim\limits_{x \to \infty} (\sqrt{x + \sqrt{x}} - \sqrt{x})$.

**Q14.** Evaluate: $\lim\limits_{x \to \infty} \{x - \sqrt{x^2 + x}\}$.

**Q15.** Evaluate: $\lim\limits_{x \to 0} \frac{\sqrt{x+1} - 1}{\log(x+1)}$.

**Q16.** Evaluate: $\lim\limits_{x \to 0} \frac{8x\sqrt{a^2 - (a-x)^2}}{(\sqrt{8ax - 4x^2} + \sqrt{8ax})^3}$.

**Q17.** The value of $\lim\limits_{x \to \infty} (\sqrt{x^2 + ax + b} - x)$.

**Q18.** Evaluate: $\lim\limits_{x \to \infty} x\sqrt{x}(\sqrt{x^3 + 1} - \sqrt{x^3 - 1})$.

**Q19.** Evaluate: $\lim\limits_{x \to \infty} (\sqrt{16x^2 + x - } \ 4x)$.

**Q20.** Evaluate: $\lim_{x \to \infty} x^2(\sqrt{x^4+1} - \sqrt{x^4-1})$.

**Q21.** Evaluate

$$\lim_{x \to 3} \frac{\sqrt{3x+7}-4}{\sqrt{x+1}-2}.$$

**Q22.** Evaluate: $\lim_{x \to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$.

**Q23.** Evaluate: $\lim_{x \to 0} \frac{(\sqrt{1+x^n} - \sqrt{1-x^n})}{x^n}$.

**Q24.** Evaluate: $\lim_{x \to 3} \left\{ \frac{x-3}{\sqrt{x-2}-\sqrt{4-x}} \right\}$.

**Q25.** Evaluate: $\lim_{x \to 0} \frac{\sqrt{x+4}-2}{\sin 5x}$.

**Q26.** Evaluate: $\lim_{x \to 7} \frac{2-\sqrt{x-3}}{x^2-49}$.

**Q27.** Evaluate: $\lim_{x \to \infty} \left[ \sqrt{x+\sqrt{x+\sqrt{x}}} - \sqrt{x} \right]$.

Date: 12/10/2021

**S1.** $\lim_{x \to \infty} \sqrt{x}(\sqrt{x+a} - \sqrt{x})$

$$= \lim_{x \to \infty} \frac{\sqrt{x}(\sqrt{x+a} - \sqrt{x})(\sqrt{x+a} + \sqrt{x})}{(\sqrt{x+a} + \sqrt{x})}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x}(x+a-x)}{\sqrt{x}\left(\sqrt{1+\frac{a}{x}}+1\right)} = \lim_{x \to \infty} \frac{a}{\left(\sqrt{1+\frac{a}{x}}+1\right)} = \frac{a}{\sqrt{1+0}+1} = \frac{a}{2}.$$

**S2.** Given, $\lim_{x\to 1} \frac{G(x)-G(1)}{x-1}\left[\text{form}\frac{0}{0}\right] = \lim_{x\to 1} \frac{G'(x)}{1} = G'(1).$

Now, $$G'(x) = -\frac{1}{2}(25-x^2)^{-1/2}(-2x) = \frac{x}{\sqrt{25-x^2}}.$$

$\therefore$ $$\text{Given limit} = G'(1) = \frac{1}{\sqrt{24}} = \frac{1}{2\sqrt{6}}.$$

**S3.**

$$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \to 0} \frac{\left(\sqrt{x+h} - \sqrt{x}\right)}{h} \cdot \frac{\left(\sqrt{x+h} + \sqrt{x}\right)}{\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{x \to 0} \frac{x+h-x}{h} \cdot \frac{1}{\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \frac{1}{2\sqrt{x}}.$$

**S4.**

$$\lim_{x\to\infty} x[\sqrt{x^2+6} - x] = \lim_{x\to\infty} \frac{x(\sqrt{x^2+6} - x)(\sqrt{x^2+6} + x)}{(\sqrt{x^2+6} + x)}$$

$$= \lim_{x\to\infty} \frac{x(x^2+6-x^2)}{\left(\sqrt{x^2+6}+x\right)} = \lim_{x\to\infty} \frac{6x}{\left(\sqrt{1+\frac{6}{x^2}}+1\right)}$$

$$= \lim_{x\to\infty} \frac{6x}{\left(\sqrt{1+\frac{6}{x^2}}+1\right)} = \frac{6}{2} = 3.$$

**S5.** $\lim\limits_{x\to 0}\left\{\dfrac{\sqrt{a+x}-\sqrt{a-x}}{x}\right\}$

$$= \lim_{x\to 0}\left\{\frac{\sqrt{a+x}-\sqrt{a-x}}{x}\cdot\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}\right\}$$

$$= \lim_{x\to 0}\frac{(a+x)-(a-x)}{x}\cdot\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$$

$$= \lim_{x\to 0}\frac{(a+x)-(a-x)}{(\sqrt{a+x}+\sqrt{a-x})}\lim_{x\to 0}\frac{2x}{(\sqrt{a+x}+\sqrt{a-x})}$$

$$= \lim_{x\to 0}\frac{2}{(\sqrt{a+x}+\sqrt{a-x})}$$

$$= \frac{2}{\sqrt{a+0}+\sqrt{a-0}}=\frac{2}{2\sqrt{a}}=\frac{1}{\sqrt{a}}.$$

**S6.** Given limit

$$= \lim_{x\to a}\frac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{3a+x}+2\sqrt{x})(\sqrt{a+2x}+\sqrt{3x})}{(\sqrt{a+2x}+\sqrt{3x})(\sqrt{3a+x}-2\sqrt{x})(\sqrt{3a+x}+2\sqrt{x})}$$

$$= \lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})3(a-x)}$$

$$= \lim_{x\to a}\frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x})3(a-x)}=\frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3a}}=\frac{2}{3\sqrt{3}}.$$

**S7.** Given limit

$$= \lim_{x\to 2}\frac{\left(\sqrt{1+\sqrt{2+x}}-\sqrt{3}\right)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)}{(x-2)\left(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\right)}$$

$$= \lim_{x\to 2}\frac{(\sqrt{2+x}-2)}{(x-2)\sqrt{1+\sqrt{2+x}}+\sqrt{3})}\times\frac{(\sqrt{2+x}+2)}{(\sqrt{2+x}+2)}$$

$$= \lim_{x\to 2}\frac{1}{(\sqrt{1+\sqrt{2+x}}+\sqrt{3}\,)(\,\sqrt{2+x}+2\,)}$$

$$= \frac{1}{2\sqrt{3} \times 4} = \frac{1}{8\sqrt{3}}.$$

**S8.** $$\lim_{x \to 2} \frac{\sqrt{x-2} + \sqrt{x} - \sqrt{2}}{\sqrt{x^2 - 4}} = \lim_{x \to 2} \left\{ \frac{\sqrt{x-2}}{\sqrt{x^2-4}} + \frac{\sqrt{x} - \sqrt{2}}{\sqrt{x^2 - 4}} \right\}$$

$$= \lim_{x \to 2} \left\{ \frac{1}{\sqrt{x+2}} + \frac{\sqrt{x} - \sqrt{2}}{\sqrt{x^2 - 4}} \cdot \frac{\sqrt{x} + \sqrt{2}}{\sqrt{x} + \sqrt{2}} \right\}$$

$$= \lim_{x \to 2} \left\{ \frac{1}{\sqrt{x+2}} + \frac{x-2}{\sqrt{x^2-4} \cdot (\sqrt{x} + \sqrt{2})} \right\} = \frac{1}{2} + 0 = \frac{1}{2}.$$

**S9.** $$\lim_{x \to 2a} \frac{\sqrt{x-2a} + \sqrt{x} - \sqrt{2a}}{(\sqrt{x^2 - 4a^2}}$$

$$= \lim_{x \to 2a} \frac{\sqrt{x-2a} + \dfrac{(\sqrt{x} - \sqrt{2a})(\sqrt{x} + \sqrt{2a})}{(\sqrt{x} + \sqrt{2a})}}{(\sqrt{x^2 + 2a}\sqrt{x-2a}}$$

$$= \lim_{x \to 2a} \left\{ \frac{\sqrt{x-2a} + \dfrac{(x-2a)}{(\sqrt{x} + \sqrt{2a})}}{(\sqrt{x^2 - 2a}\sqrt{x+2a}} \right\}$$

$$= \lim_{x \to 2a} \left\{ \frac{1}{\sqrt{x+2a}} + \frac{\sqrt{x-2a}}{(\sqrt{x+2a})(\sqrt{x} + \sqrt{2a})} \right\}$$

$$= \frac{1}{\sqrt{4a}} + \frac{0}{\sqrt{4a}(2\sqrt{2a})} = \frac{1}{2\sqrt{a}}.$$

**S10.** $$\lim_{x \to 2} \frac{\sqrt{x^2 + x - 3} - \sqrt{x+1}}{(x-2)}$$

$$= \lim_{x \to 2} \frac{(\sqrt{x^2 + x - 3} - \sqrt{x+1})(\sqrt{x^2 + x - 3} + \sqrt{x+1})}{(x-2)(\sqrt{x^2 + x - 3} + \sqrt{x+1})}$$

$$= \lim_{x \to 2} \frac{(x^2 - 4)}{(x-2)(\sqrt{x^2 + x - 3} + \sqrt{x+1})}$$

$$= \lim_{x \to 2} \frac{(x+2)}{(\sqrt{x^2 + x - 3} + \sqrt{x+1})} = \frac{4}{\sqrt{3} + \sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2\sqrt{3}}{3}.$$

**S11.** $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}$

$$= \lim_{x\to 0} \frac{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)(\sqrt{x^2+9}+3)}{(\sqrt{x^2+9}-3)(\sqrt{x^2+1}+1)(\sqrt{x^2+9}+3)}$$

$$= \lim_{x\to 0} \frac{\{(x^2+1)-1\}\sqrt{x^2+9}+3}{\{(x^2+9)-9\})(\sqrt{x^2+1}+1)} = \lim_{x\to 0} \frac{x^2\sqrt{x^2+9}+3}{x^2(\sqrt{x^2+1}+1)}$$

$$= \lim_{x\to 0} \frac{(\sqrt{x^2+9}+3)}{(\sqrt{x^2+1}+1)} = \frac{3+3}{1+1} = \frac{6}{2} = 3.$$

**S12.** $\lim\limits_{x\to\infty} (\sqrt{x^2+2x-1}-x)$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+2x-1}-x)(\sqrt{x^2+2x+1}+x)}{(\sqrt{x^2+2x-1}+x)}$$

$$= \lim_{x\to\infty} \frac{(x^2+2x-1)-x^2}{\sqrt{x^2+2x-1}+x} = \lim_{x\to\infty} \frac{\left(2-\frac{1}{x}\right)}{\left(\sqrt{1+\frac{2}{x}-\frac{1}{x^2}}+1\right)}$$

$$= \frac{2-0}{\sqrt{1+0-0}+1} = \frac{2}{2} = 1.$$

**S13.**

$$\lim_{x\to\infty} (\sqrt{x+\sqrt{x}}-\sqrt{x}) = \lim_{x\to\infty} \frac{(\sqrt{x+\sqrt{x}}-\sqrt{x})(\sqrt{x+\sqrt{x}}+\sqrt{x})}{(\sqrt{x+\sqrt{x}}+\sqrt{x})}$$

$$= \lim_{x\to\infty} \frac{(x+\sqrt{x})-x}{(\sqrt{x+\sqrt{x}}+\sqrt{x})} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{1+\sqrt{\frac{1}{x}}}+1\right)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x}}{\left(\sqrt{1+\sqrt{\frac{1}{x}}}+1\right)} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}.$$

**S14.**

$$\lim_{x\to\infty} (x-\sqrt{x^2+x}) = \lim_{x\to\infty} (x-\sqrt{x^2+x}) \cdot \frac{(x+\sqrt{x^2+x})}{(x+\sqrt{x^2+x})}$$

$$= \lim_{x\to\infty} (x-\sqrt{x^2+x}) \cdot \frac{(x+\sqrt{x^2+x})}{(x+\sqrt{x^2+x})}$$

$$= \lim_{x \to \infty} \frac{x^2 - (x^2 + x)}{(x + \sqrt{x^2 + x})} = \lim_{x \to \infty} \frac{-x}{x\left(1 + \sqrt{1 + \frac{1}{x}}\right)}$$

$$= \lim_{x \to \infty} \frac{-x}{\left(1 + \sqrt{1 + \frac{1}{x}}\right)} = \frac{-1}{2}.$$

**S15.**

$$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{\log(x+1)} = \lim_{x \to 0} \frac{(\sqrt{x+1} - 1)}{\log(x+1)} \cdot \frac{(\sqrt{x+1} + 1)}{(\sqrt{x+1} + 1)}$$

$$= \lim_{x \to 0} \frac{(x+1) - 1}{\log(x+1)} \cdot \frac{1}{(\sqrt{x+1} + 1)}$$

$$= \lim_{x \to 0} \frac{x}{\log(x+1)} \cdot \lim_{x \to 0} \frac{1}{(\sqrt{x+1} + 1)}$$

$$= \frac{1}{\lim_{x \to 0} \frac{\log(1+x)}{x}} \cdot \lim_{x \to 0} \frac{1}{(\sqrt{x+1} + 1)} = \frac{1}{1} \cdot \frac{1}{(1+1)} = \frac{1}{2}.$$

**S16.**

$$\lim_{x \to \infty} \frac{8x\sqrt{a^2 - (a-x)^2}}{(\sqrt{8ax - 4x^2} + \sqrt{8ax})^3} = \lim_{x \to 0} \frac{8x\sqrt{2ax - x^2}}{8x^{3/2}(\sqrt{2a - x} + \sqrt{2a})^3}$$

$$= \lim_{x \to 0} \frac{x^{3/2}\sqrt{2a - x}}{x^{3/2}(\sqrt{2a - x} + \sqrt{2a})^3}$$

$$= \lim_{x \to 0} \frac{\sqrt{2a - x}}{(\sqrt{2a - x} + \sqrt{2a})^3} = \frac{\sqrt{2a}}{(\sqrt{2a} + \sqrt{2a})^3}$$

$$= \frac{\sqrt{2a}}{(2 + \sqrt{2a})^3} = \frac{1}{16a}.$$

**S17.**

$$\lim_{x \to \infty} \frac{(\sqrt{x^2 + ax + b} - x)(\sqrt{x^2 + ax + b} + x)}{(\sqrt{x^2 + ax + b} + x)}$$

$$= \lim_{x \to \infty} \frac{(x^2 + ax + b) - x^2}{(\sqrt{x^2 + ax + b} + x)} = \lim_{x \to \infty} \left\{ \frac{ax + b}{\sqrt{x^2 + ax + b} + x} \right\}$$

$$= \lim_{x \to \infty} \frac{a\left(a + \frac{b}{x}\right)}{x\left\{\sqrt{1 + \frac{a}{x} + \frac{b}{x^2}} + 1\right\}}$$

$$= \lim_{x \to \infty} \frac{a\left(a + \frac{b}{x}\right)}{x\left\{\sqrt{1 + \frac{a}{x} + \frac{b}{x^2}} + 1\right\}} = \frac{a + 0}{\sqrt{1+0} + 1} = \frac{a}{2}.$$

**S18.** $\lim_{x \to \infty} x\sqrt{x}(\sqrt{x^3 + 1} - \sqrt{x^3 - 1})$

$$= \lim_{x \to \infty} \frac{x\sqrt{x}(\sqrt{x^3+1} - \sqrt{x^3 - 1}).(\sqrt{x^3+1} + \sqrt{x^3-1})}{(\sqrt{x^3+1} + \sqrt{x^3-1})}$$

$$= \lim_{x \to \infty} \frac{x^{3/2}\{(x^3+1) - (x^3-1)\}}{x^{3/2}\left\{\sqrt{1 + \frac{1}{x^3}} + \sqrt{1 - \frac{1}{x^3}}\right\}}$$

$$= \lim_{x \to \infty} \frac{2}{x^{3/2}\left\{\sqrt{1 + \frac{1}{x^3}} + \sqrt{1 - \frac{1}{x^3}}\right\}} = \frac{2}{\sqrt{1-0} + \sqrt{1-0}}$$

$$= \frac{2}{1+1} = \frac{2}{2} = 1.$$

**S19.** $\lim_{x \to \infty} (\sqrt{16x^2 + x} - 4x)$

$$= \lim_{x \to \infty} \frac{(\sqrt{16x^2+x} - 4x)(\sqrt{16x^2+x} + 4x)}{(\sqrt{16x^2+x} + 4x)}$$

$$= \lim_{x \to \infty} \frac{(\sqrt{16x^2+x} - 4x)(\sqrt{16x^2+x} + 4x)}{(\sqrt{16x^2+x} + 4x)}$$

$$= \lim_{x \to \infty} \frac{(16x^2 + x) - 16x^2}{(\sqrt{16x^2+x} + 4x)}$$

$$= \lim_{x \to \infty} \frac{x}{\left(\sqrt{16 + \frac{1}{x}} + 4\right)} = \lim_{x \to \infty} \frac{1}{\left(\sqrt{16 + \frac{1}{x}} + 4\right)}$$

$$= \frac{1}{\sqrt{16+0} + 4} = \frac{1}{4+4} = \frac{1}{8}.$$

**S20.** $\lim_{x \to \infty} x^2(\sqrt{x^4 + 1} - \sqrt{x^4 - 1})$

$$= \lim_{x \to \infty} \frac{x^2(\sqrt{x^4+1} - \sqrt{x^4-1})(\sqrt{x^4+1} + \sqrt{x^4-1})}{(\sqrt{x^4+1} + \sqrt{x^4-1})}$$

$$= \lim_{x \to \infty} \frac{x^2\{(x^4+1)-(x^4-1)\}}{(\sqrt{x^4+1}+\sqrt{x^4-1})}$$

$$= \lim_{x \to \infty} \frac{2x^2}{x^2\left(\sqrt{1+\frac{1}{x^4}}+\sqrt{1-\frac{1}{x^4}}\right)}$$

$$= \lim_{x \to \infty} \frac{2x^2}{x^2\left(\sqrt{1+\frac{1}{x^4}}+\sqrt{1-\frac{1}{x^4}}\right)} = \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{2} = 1.$$

**S21.**

$$\lim_{x \to 3} \frac{\sqrt{3x+7}-4}{\sqrt{x+1}-2} = \lim_{x \to 3} \frac{(\sqrt{3x+7}-4)(\sqrt{3x+7}+4)(\sqrt{x+1}+2)}{(\sqrt{x+1}-2)(\sqrt{x+1}+2)(\sqrt{3x+7}+4)}$$

$$= \lim_{x \to 3} \left\{\frac{(3x+7)-16}{(x-1)-4} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{3x+7}+4)}\right\}$$

$$= \lim_{x \to 3} \left\{\frac{(3x-9)}{(x-3)} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{3x+7}+4)}\right\}$$

$$= 3 \lim_{x \to 3} \left\{\frac{(\sqrt{x+1}+2)}{\sqrt{3x+7}+4}\right\} = 3 \cdot \frac{(2+2)}{(4+4)} = 3 \times \frac{4}{8} = \frac{3}{2}.$$

**S22.**

$$\lim_{x \to 4} \frac{3-\sqrt{5+x}}{1-\sqrt{5-x}} = \lim_{x \to 4} \frac{(3-\sqrt{5+x})(3+\sqrt{5+x})(1+\sqrt{5-x})}{(1-\sqrt{5-x})(1+\sqrt{5-x})(3+\sqrt{5+x})}$$

$$= \lim_{x \to 4} \frac{9-(5+x)}{1-(5-x)} \cdot \frac{(1+\sqrt{5-x})}{(3+\sqrt{5+x})} = \lim_{x \to 4} \frac{(4-x)}{(x-4)} \cdot \frac{(1+\sqrt{5-x})}{(3+\sqrt{5+x})}$$

$$= -\lim_{x \to 4} \frac{(1+\sqrt{5-x})}{(3+\sqrt{5+x})} = -\left(\frac{1+1}{3+3}\right) = -\frac{2}{6} = -\frac{1}{3}.$$

**S23.**

$$\lim_{x \to 0} \frac{(\sqrt{1+x^n}-\sqrt{1-x^n})}{x^n}$$

$$= \lim_{x \to 0} \frac{(\sqrt{1+x^n}-\sqrt{1-x^n})(\sqrt{1+x^n}+\sqrt{1+x^n})}{x^n(\sqrt{1+x^n}+\sqrt{1-x^n})}$$

$$= \lim_{x \to 0} \frac{(1+x^n)-(1-x^n)}{x^n(\sqrt{1+x^n}+\sqrt{1-x^n})} = \lim_{x \to 0} \frac{2x^n}{x^n(\sqrt{1+x^n}+\sqrt{1-x^n})}$$

$$= \lim_{x \to 0} \frac{2}{(\sqrt{1+x^n} + \sqrt{1-x^n})} = \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = \frac{2}{2} = 1.$$

**S24.** $\lim_{x \to 3} \left[ \frac{x-3}{\sqrt{x-2} - \sqrt{4-x}} \right]$

$$= \lim_{x \to 3} \frac{(x-3)(\sqrt{x-2} + \sqrt{4-x})}{(\sqrt{x-2} - \sqrt{4-x})(\sqrt{x-2} + \sqrt{4-x})}$$

$$= \lim_{x \to 3} \frac{(x-3)(\sqrt{x-2} + \sqrt{4-x})}{(x-2)-(4-x)}$$

$$= \lim_{x \to 3} \frac{(x-3)(\sqrt{x-2} + \sqrt{4-x})}{(x-2)-(4-x)}$$

$$= \lim_{x \to 3} \frac{\sqrt{x-2} + \sqrt{4-x}}{2} = \frac{\sqrt{1} + \sqrt{1}}{2} = \frac{2}{2} = 1.$$

**S25.**

$$\frac{\sqrt{x+4} - 2}{\sin 5x} = \lim_{x \to 0} \frac{(\sqrt{x+4} - 2)(\sqrt{x+4} + 2)}{(\sin 5x)(\sqrt{x+4} + 2)}$$

$$= \lim_{x \to 0} \frac{\{(x+4) - 4\}}{(\sin 5x)(\sqrt{x+4} + 2)}$$

$$= \lim_{x \to 0} \frac{x}{\sin 5x} \cdot \lim_{x \to 0} \frac{1}{(\sqrt{x+4} + 2)}$$

$$= \frac{1}{5} \left\{ \lim_{x \to 0} \frac{5x}{\sin 5x} \right\} \cdot \left\{ \lim_{x \to 0} \frac{1}{(\sqrt{x+4} + 2)} \right\}$$

$$= \frac{1}{5} \times 1 \times \frac{1}{(\sqrt{0+4} + 2)} = \frac{1}{5 \times 4} = \frac{1}{20}.$$

**S26.**

$$\lim_{x \to 7} \frac{2 - \sqrt{x-3}}{x^2 - 49} = \lim_{x \to 7} \frac{(2 - \sqrt{x-3})(2 + \sqrt{x-3})}{(x^2 - 49)(2 + \sqrt{x-3})}$$

$$= \lim_{x \to 7} \frac{\{4 - (x-3)\}}{(x-7)(x+7)(2 + \sqrt{x-3})}$$

$$= \lim_{x \to 7} \frac{-(x-7)}{(x-7)(x+7)(2 + \sqrt{x-3})} = \lim_{x \to 7} \frac{-1}{(x+7)(2 + \sqrt{x-3})}$$

$$= \frac{-(x-7)}{(7-7)(2 + \sqrt{7-3})} = \frac{-1}{56}.$$

**S27.** $\lim\limits_{x\to\infty}\left[\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right]$

$$= \lim_{x\to\infty}\frac{(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x})(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x})}{(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x})}$$

$$= \lim_{x\to\infty}\frac{x+\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x})(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x})}{(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x})}$$

$$= \lim_{x\to\infty}\frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}} = \lim_{x\to\infty}\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}$$

$$= \lim_{x\to\infty}\frac{\sqrt{x+x^{1/2}}}{\sqrt{x+\sqrt{x+x^{1/2}}}+\sqrt{x}}$$

$$= \lim_{x\to\infty}\frac{\sqrt{x}(\sqrt{x+x^{-1/2}})}{\sqrt{x}\,(\sqrt{1+\sqrt{x^{-1}+x^{-3/2}}}+1)}$$

$$= \lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x^{1/2}}}}{\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x^{3/2}}}}+1} = \frac{\sqrt{1+0}}{\sqrt{1+\sqrt{0}}+1} = \frac{1}{2}$$

Date: 12/10/2021

**Q1.** Evaluate the limit: $\lim_{x \to 0} \frac{\sin ax}{bx}$.

**Q2.** Evaluate: $\lim_{x \to 0} \frac{\tan^{-1}(2x)}{\sin 3x}$

**Q3.** Evaluate: $\lim_{x \to 0} \left\{\frac{1 - \cos 2x}{\sin^2 2x}\right\}$

**Q4.** Evaluate: $\lim_{x \to 0} \frac{\sin x^2}{x}$.

**Q5.** Prove that $\lim_{x \to 0} \frac{\tan x}{x} = 1$.

**Q6.** Find the value of $\lim_{x \to 0} \frac{\tan 8x}{\sin 2x}$.

**Q7.** Find the value of $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$.

**Q8.** Evaluate the limit: $\lim_{x \to 0} \frac{\cos x}{\pi - x}$.

**Q9.** Evaluate the limit: $\lim_{x \to 0} x \sec x$.

**Q10.** Evaluate: $\lim_{x \to 0} x^2 \cdot \sin \frac{\pi}{x}$

**Q11.** Evaluate: $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$.

**Q12.** Evaluate: $\lim_{x \to 0} \frac{\sin x^\circ}{x}$

**Q13.** Evaluate: $\lim_{\theta \to 0} \frac{1 - \cos\theta}{2\theta^2}$

**Q14.** Evaluate: $\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$.

**Q15.** Evaluate: $\lim_{x \to 0} \frac{\sin(x + 2) + \sin(x - 2)}{x}$.

**Q16.** Evaluate: $\lim_{x \to 0} \frac{\sin(3 - x) - \sin(3 + x)}{x}$.

**Q17.** Evaluate: $\lim_{x \to 0} x \cdot \sin \frac{1}{x}$.

**Q18.** Evaluate: $\lim_{x \to 0} \frac{\sin^2 5x}{x^2}$.

**Q19.** Evaluate: $\lim_{x \to 0} \frac{ax + x\cos x}{b \sin x}$.

**Q20.** Evaluate: $\lim_{x \to \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2}$.

Q21. Find the value of $\lim_{x \to 0} \frac{\tan x - \sin x}{\sin^3 x}$.

Q22. Evaluate: $\lim_{x \to \pi} \frac{1+\cos x}{\tan^2 x}$.

Q23. Evaluate: $\lim_{x \to 0} \frac{\cos Ax - \cos Bx}{x^2}$.

Q24. Evaluate: $\lim_{x \to 0} \frac{1-\cos mx}{1-\cos nx}$.

Q25. Find the value of $\lim_{x \to 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x}$.

Q26. Evaluate the following limits: $\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx}$.

Q27. Evaluate the following limits: $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$.

Q28. Evaluate: $\lim_{x \to 0} \frac{\sin 4x}{\sin 2x}$.

Q29. Evaluate: $\lim_{x \to 0} \frac{\sin 2x + 3x}{4x - \sin 5x}$.

Q30. Evaluate: $\lim_{x \to 0} \frac{\tan x - \sin x}{x(1-\cos 2x)}$.

Q31. Find the value of $\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx}$, $a$, $b$, $(a + b) \neq 0$.

Q32. Evaluate the following limits: $\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$.

Q33. Evaluate: $\lim_{y \to 0} \frac{(x+y)\cdot \sec(x+y) - x \sec x}{y}$.

Q34. Evaluate: $\lim_{x \to \frac{\pi}{4}} \frac{1-\tan x}{\left(x - \frac{\pi}{4}\right)}$.

Q35. Find the value of $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$.

Q36. Evaluate: $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$.

Q37. Evaluate: $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sin^{-1} x}$.

Date: 12/10/2021

**S1.** We have,

$$\lim_{x \to 0} \frac{\sin ax}{bx} = \lim_{x \to 0} \left[\frac{\sin ax}{ax} \cdot \frac{ax}{bx}\right]$$

$$= \frac{a}{b}\left[\lim_{x \to 0} \frac{\sin ax}{ax}\right] = \frac{a}{b}.$$

**S2.** Given

$$\lim_{x \to 0} \frac{\tan^{-1}(2x)}{\sin 3x} = \lim_{x \to 0} \frac{\left(\frac{\tan^{-1} 2x}{2x}\right) \cdot 2x}{\frac{\sin 3x}{3x} \times 3x}$$

$\therefore$ $$= \frac{2x}{3x} \times \frac{1}{1} = \frac{2}{3}.$$

**S3.** Given

$$\lim_{x \to 0} \left\{\frac{1 - \cos 2x}{\sin^2 2x}\right\} = \lim_{x \to 0} \frac{2\sin^2 x}{4 \cdot \sin^2 x \cdot \cos^2 x}$$

$\therefore$ $$= \frac{1}{2} \lim_{x \to 0} \frac{1}{\cos^2 x} = \frac{1}{2}.$$

**S4.**

$$\lim_{x \to 0} \frac{\sin x^2}{x} = \lim_{x \to 0} \frac{\sin x^2}{x^2} \cdot \lim_{x \to 0} x$$

$$= 1 \times 0 = 0.$$

**S5.**

$$\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1}{\cos x}$$

$$= 1 \times 1 = 1.$$

**S6.**

$$\lim_{x \to 0} \frac{\tan 8x}{\sin 2x} = \left\{\lim_{x \to 0} \frac{\tan 8x}{8x} \times 8x \cdot \lim_{x \to 0} \frac{2x}{\sin 2x} \cdot \frac{1}{2x}\right\}$$

$$= 8x \times 1 \times \frac{1}{2x} = 4.$$

**S7.** $\because$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\therefore \qquad \lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{\frac{\sin ax}{ax} \times ax}{\frac{\sin bx}{bx} \times bx}$$

$$\therefore \qquad \lim_{x \to 0} \frac{1 \times ax}{1 \times bx} = \frac{a}{b} = \frac{a}{b}.$$

**S8.** We have, $\lim_{x \to 0} \frac{\cos x}{\pi - x} = \frac{1}{\pi - 0} = \frac{1}{\pi}.$

**S9.** We have, $\lim_{x \to 0} x \sec x = \lim_{x \to 0} \frac{x}{\cos x} = \frac{0}{1} = 0.$

**S10.**

$$\lim_{x \to 0} x^2 \cdot \sin \frac{\pi}{x} = \lim_{x \to 0} \frac{\sin \frac{\pi}{x}}{\frac{1}{x^2}}$$

since, $-1 \le \sin \frac{\pi}{x} \le 1$

$\because$ as $x \to 0$, $\frac{1}{x^2} \to \infty$

Hence, $\lim_{x \to 0} x^2 \sin \frac{\pi}{x} = 0$

(It should be noted that limit does not gives exact value, it yields only limiting value.)

**S11.**

$$\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)} = \frac{1}{\pi} \lim_{x \to \pi} \frac{\sin(\pi - x)}{(\pi - x)}$$

$$= \frac{1}{\pi} \lim_{x \to \pi} \frac{\sin h}{h} \qquad \left\{ \begin{array}{ll} \because & x \to \pi \\ \Rightarrow & \pi - x \to h \end{array} \right\}$$

$$= \frac{1}{\pi} \times 1 = \frac{1}{\pi}.$$

**S12.** $\because$ $\pi$ radian = 180º

$\therefore$ $x^\circ = \frac{\pi x}{180}$ radian

So

$$\lim_{x \to 0} \frac{\sin x^\circ}{x} = \lim_{x \to 0} \frac{\sin x \cdot \frac{\pi}{180^\circ}}{x \cdot \frac{\pi}{180^\circ}} \times \frac{\pi}{180^\circ}$$

$$= \frac{\pi}{180^\circ}.$$

**S13.** Given

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{2\theta^2} = \lim_{\theta \to 0} \frac{2\sin\frac{\theta}{2}}{2\theta^2}$$

$\therefore$
$$= 2\lim_{\theta \to 0}\left(\frac{\sin\frac{\theta}{2}}{\frac{\theta}{2}}\right)^2 \times \frac{1}{8}$$

So
$$= 2 \times \frac{1}{8} = \frac{1}{4}.$$

**S14.** Given, $\lim_{x \to a} \frac{\sin x - \sin a}{\sqrt{x} - \sqrt{a}}$

$$\Rightarrow \lim_{x \to a} \frac{2\cos\left\{\frac{x+a}{2}\right\} \cdot \sin\left\{\frac{x-a}{2}\right\}}{\sqrt{x} - \sqrt{a}} = \lim_{x \to a} \cos\left\{\frac{x+a}{2}\right\} \cdot \lim_{x \to a} \frac{\sin\left\{\frac{x+a}{2}\right\}}{\left\{\frac{x+a}{2}\right\}} \cdot \lim_{x \to a}\left(\sqrt{x} + \sqrt{a}\right)$$

$$= (\cos a) \times 1 \times 2\sqrt{a} = 2\sqrt{a} \cdot \cos a.$$

**S15.**

Given, $$\lim_{x \to 0} \frac{\sin(x+2) + \sin(x-2)}{x} = \lim_{x \to 0} \frac{2\sin\left(\frac{(x+2)+(x-2)}{2}\right) \cdot \cos\left(\frac{(x+2)-(x-2)}{2}\right)}{x}$$

$$= \lim_{x \to 0} 2\frac{\sin x}{x} \cdot \cos 2 = 2\cos 2 \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 2\cos 2.$$

**S16.**

$$\lim_{x \to 0} \frac{\sin(3-x) - \sin(3+x)}{x} = \lim_{x \to 0} \frac{2\cos\left(\frac{(3-x)+(3+x)}{2}\right) \cdot \sin\left(\frac{(3-x)-(3+x)}{2}\right)}{x}$$

$$= -2\cos 3 \lim_{x \to 0} \frac{\sin x}{x}$$

$$= -2\cos 3.$$

**S17.** $\because \quad -1 \le \sin\frac{1}{x} \le 1 \quad$ for $\quad x \in R$

$\therefore$
$$\lim_{x \to 0} x \cdot \sin\frac{1}{x} = \lim_{x \to 0} \frac{\sin\frac{1}{x}}{\frac{1}{x}}$$

As $x \to 0, \quad \frac{1}{x} \to \infty.$

Hence, $$\lim_{x \to 0} x \sin \frac{1}{x} = 0.$$

**S18.**

$$\lim_{x \to 0} \frac{\sin^2 5x}{x^2} = \lim_{x \to 0} \frac{\sin 5x}{x} \cdot \frac{\sin 5x}{x}$$

$$= \lim_{x \to 0} 5 \cdot \left(\frac{\sin 5x}{5x}\right) \times 5 \left(\frac{\sin 5x}{5x}\right)$$

$$= 5 \cdot \left(\lim_{x \to 0} \frac{\sin 5x}{5x}\right) \cdot 5 \cdot \left(\lim_{x \to 0} \frac{\sin 5x}{5x}\right)$$

$$= 5 \times 1 \times 5 \times 1 = 25.$$

**S19.**

$$\lim_{x \to 0} \frac{ax + x \cos x}{b \sin x} = \lim_{x \to 0} \left(\frac{ax}{b \sin x} + \frac{x \cos x}{b \sin x}\right)$$

$$= \frac{a}{b} \cdot \lim_{x \to 0} \frac{x}{\sin x} + \frac{1}{b} \lim_{x \to 0} \frac{x \cos x}{\sin x}$$

$$= \frac{a}{b} \times 1 + \frac{1}{b} \lim_{x \to 0} \frac{x}{\tan x}$$

$$= \frac{a}{b} + \frac{1}{b} = \frac{a+1}{b}$$

**S20.**

$$\lim_{x \to \frac{\pi}{2}} \frac{1 + \cos 2x}{(\pi - 2x)^2} = \lim_{h \to 0} \frac{1 + \cos 2\left(\frac{\pi}{2} + h\right)}{\left[\pi - 2\left(\frac{\pi}{2} + h\right)\right]^2}$$

Put $\left(x = \frac{\pi}{2} + h\right)$ as $x \to \frac{\pi}{2}, h \to 0$

$$= \lim_{h \to 0} \frac{1 + \cos(\pi + 2h)}{(\pi - \pi - 2h)^2} = \lim_{h \to 0} \frac{1 + \cos 2h}{4h^2}$$

$$= \lim_{h \to 0} \frac{1 - (1 - 2\sin^2 h)}{4h^2} = \lim_{h \to 0} \frac{2\sin^2 h}{4h^2}$$

$$= \frac{1}{2} \lim_{h \to 0} \left(\frac{\sin h}{h}\right)^2$$

$$= \frac{1}{2} \left(\lim_{h \to 0} \frac{\sin h}{h}\right) \left(\lim_{h \to 0} \frac{\sin h}{h}\right)$$

$$= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}.$$

**S21.**

$$\lim_{x \to 0} \frac{\tan x - \sin x}{\sin^3 x} = \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x}$$

$$= \lim_{x \to 0} \frac{\sin x - \sin x \cos x}{\sin^3 x \cdot \cos x}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x \cdot \cos x}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{(1 - \cos x)(\cos x + 1)(\cos x)}$$

$\because$ $\sin^2 x = 1 - \cos^2 x$

$$= \lim_{x \to 0} \frac{1}{\cos x (1 + \cos x)} = 1 \times \frac{1}{2} = \frac{1}{2}.$$

**S22.**

$$\lim_{x \to \pi} \frac{1 + \cos x}{\tan^2 x} = \lim_{x \to \pi} \frac{2 \cos^2 \frac{x}{2}}{\sin^2 x} \times \cos^2 x$$

$\because$ $\tan^2 x = \dfrac{\sin^2 x}{\cos^2 x}$

$$= \lim_{x \to \pi} \frac{2 \cos^2 \frac{x}{2}}{4 \sin^2 \frac{x}{2} \cdot \cos^2 \frac{x}{2}} \times (\cos^2 x)$$

$$= \lim_{x \to \pi} \frac{\cos^2 x}{2 \sin^2 \frac{x}{2}} = \frac{\cos^2 \pi}{2 \sin^2 \frac{\pi}{2}} = \frac{1}{2}.$$

**S23.**

Given, $$\lim_{x \to 0} \frac{\cos Ax - \cos Bx}{x^2} = \lim_{x \to 0} \frac{2 \sin \frac{Ax + Bx}{2} \cdot \sin \frac{Bx - Ax}{2}}{x^2}$$

$$= \lim_{x \to 0} 2 \cdot \frac{\sin\left(\left(\frac{A+B}{2}\right) \cdot x\right)}{x} \cdot \frac{\sin\left(\left(\frac{B-A}{2}\right) \cdot x\right)}{x}$$

$$= 2 \cdot \lim_{x \to 0} \frac{\sin\left(\frac{A+B}{2}\right) x}{\left(\frac{A+B}{2}\right) x} \cdot \left(\frac{A+B}{2}\right)$$

$$\lim_{x \to 0} \frac{\sin \frac{(B-A)}{2} \cdot x}{\frac{(B-A)}{2} \cdot x} \cdot \frac{(B-A)}{2}$$

$$= 2 \times 1 \times \left(\frac{B+A}{2}\right) \times 1 \times \frac{(B-A)}{2} = \frac{B^2 - A^2}{2}.$$

**S24.**

Given, $$\lim_{x \to 0} \frac{1 - \cos mx}{1 - \cos nx} = \lim_{x \to 0} \frac{2 \sin^2 \frac{mx}{2}}{2 \sin^2 \frac{nx}{2}} = \lim_{x \to 0} \left( \frac{\sin \frac{mx}{2}}{\sin \frac{nx}{2}} \right)^2$$

$$= \lim_{x \to 0} \left[ \frac{\sin \frac{mx}{2}}{\frac{mx}{2}} \times \frac{mx}{2} \right]^2 \cdot \lim_{x \to 0} \left( \frac{\frac{nx}{2}}{\sin \frac{nx}{2}} \times \frac{1}{\frac{nx}{2}} \right)^2$$

$$= m^2 \times \frac{1}{n^2} = \frac{m^2}{n^2}.$$

**S25.**

Given, $$\lim_{x \to 0} \frac{\cot 2x - \operatorname{cosec} 2x}{x} = \lim_{x \to 0} \frac{\frac{\cos 2x}{\sin 2x} - \frac{1}{\sin 2x}}{x}$$

$$= \lim_{x \to 0} \frac{\cos 2x - 1}{x \cdot \sin 2x}$$

$$= \lim_{x \to 0} \frac{-2 \sin^2 x}{x \sin 2x}$$

$$= \lim_{x \to 0} \frac{-2 \sin^2 x}{x \cdot 2 \sin x \cdot \cos x}$$

$$= \lim_{x \to 0} \frac{-\sin x}{x \cos x} = \lim_{x \to 0} -\left(\frac{\sin x}{x}\right) \cdot \frac{1}{\cos x}$$

$$= (-1) \times 1 = -1.$$

**S26.** Let $$\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx} = \lim_{x \to 0} \left[ \frac{\sin ax}{ax + \sin bx} + \frac{bx}{ax + \sin bx} \right]$$

$$= \lim_{x \to 0} \left[ \frac{\frac{\sin ax}{ax}}{1 + \frac{\sin bx}{ax}} \right] + \lim_{x \to 0} \left[ \frac{1}{\frac{ax}{bx} + \frac{\sin bx}{bx}} \right]$$

$$= \lim_{x \to 0} \left[ \frac{\frac{\sin ax}{ax}}{1 + \frac{\sin bx}{ax} \times \frac{bx}{bx}} \right] + \lim_{x \to 0} \left[ \frac{1}{\frac{a}{b} + \frac{\sin bx}{bx}} \right]$$

$$= \left[ \frac{\lim_{ax \to 0} \frac{\sin ax}{ax}}{1 + \lim_{bx \to 0} \frac{\sin bx}{bx} \cdot \frac{b}{a}} \right] + \left[ \frac{1}{\frac{a}{b} + \lim_{bx \to 0} \frac{\sin bx}{bx}} \right]$$

$$= \left[ \frac{1}{1 + \frac{b}{a}} \right] + \left[ \frac{1}{\frac{a}{b} + 1} \right]$$

$$= \left[ \frac{a}{a+b} \right] + \left[ \frac{b}{a+b} \right] = \frac{a+b}{a+b} = 1.$$

**S27.** We have, $\lim_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1} = \lim_{x \to 0} \frac{(2\cos^2 x - 1) - 1}{\cos x - 1}$

$$= \lim_{x \to 0} \frac{2(\cos^2 x - 1)}{\cos x - 1}$$

$$= \lim_{x \to 0} 2 \left[ \frac{(\cos x + 1)(\cos x - 1)}{(\cos x - 1)} \right]$$

$$= 2 \lim_{x \to 0} (\cos x + 1)$$

$$= 2(1 + 1) = 4.$$

**S28.** Given, $\lim_{x \to 0} \frac{\sin 4x}{\sin 2x} = \lim_{x \to 0} \left[ \frac{\sin 4x}{4x} \cdot \frac{2x}{\sin 2x} \cdot 2 \right]$

$$= 2 \cdot \lim_{x \to 0} \left[ \frac{\sin 4x}{4x} \right] \div \left[ \frac{\sin 2x}{2x} \right]$$

$$= 2 \cdot \lim_{x \to 0} \left[ \frac{\sin 4x}{4x} \right] \div \lim_{x \to 0} \left[ \frac{\sin 2x}{2x} \right]$$

$$= 2.1.1 = 2.$$

**S29.** Given,

$$\lim_{x \to 0} \frac{\sin 2x + 3x}{4x - \sin 5x} = \lim_{x \to 0} \frac{2 \times \frac{\sin 2x}{2x} + \frac{3x}{x}}{\frac{4x}{x} - \frac{\sin 5x}{5x} \times 5}$$

$$= \lim_{x \to 0} \frac{2\left(\frac{\sin 2x}{2x}\right) + 3}{4 - \left(\frac{\sin 5x}{5x}\right) \times 5}$$

$$= \frac{2 \times 1 + 3}{4 - 5 \times 1} = \frac{2 + 3}{4 - 5} = -5.$$

**S30.** Given,

$$\lim_{x \to 0} \frac{\tan x - \sin x}{x(1 - \cos 2x)} = \lim_{x \to 0} \frac{\sin x \left( \frac{1}{\cos x} - 1 \right)}{x \cdot 2 \sin^2 x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \lim_{x \to 0} \frac{1 - \cos x}{2 \left( 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} \right)^2} \cdot \frac{1}{\cos x}$$

$$= \lim_{x \to 0} \frac{2 \sin^2 \frac{x}{2}}{2 \times 4 \sin^2 \frac{x}{2} \cdot \cos^2 \frac{x}{2}} \cdot \lim_{x \to 0} \frac{1}{\cos x}$$

$$= \lim_{x \to 0} \frac{1}{4 \cos^2 \frac{x}{2}} = \frac{1}{4}.$$

**S31.**

Given,
$$\lim_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx} = \lim_{x \to 0} \frac{\frac{\sin ax}{x} + \frac{bx}{x}}{\frac{ax}{x} + \frac{\sin bx}{x}}$$

$$= \lim_{x \to 0} \frac{a \cdot \sin \frac{ax}{ax} + b}{a + \left( \sin \frac{bx}{bx} \right) \cdot b}$$

$$= \frac{a \times 1 + b}{a + b \times 1} = \frac{a + b}{a + b} = 1.$$

**S32.** Put $y = x - \frac{\pi}{2}$. So, as $x \to \frac{\pi}{2}$, $y \to 0$, we have

$$\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = \lim_{y \to 0} \frac{\tan 2\left( y + \frac{\pi}{2} \right)}{y}$$

$$= \lim_{y \to 0} \frac{\tan (2y + \pi)}{y}$$

$$= \lim_{y \to 0} \frac{\tan 2y}{y} = \lim_{y \to 0} \frac{\sin 2y}{y \cos 2y}$$

$$= \lim_{y \to 0} \frac{\sin 2y}{2y} \cdot \frac{2y}{y \cos 2y}$$

$$= \left[\lim_{y \to 0} \frac{\sin 2y}{2y}\right] \cdot \left[\lim_{y \to 0} \frac{2}{\cos 2y}\right]$$

$$= 1 \cdot 2 = 2.$$

**S33.** Given, $\lim_{y \to 0} \frac{(x+y) \cdot \sec(x+y) - x \sec x}{y}$

$$= \lim_{y \to 0} \frac{\frac{x+y}{\cos(x+y)} - \frac{x}{\cos x}}{y}$$

$$= \lim_{y \to 0} \frac{(x+y) \cdot \cos x - x \cos(x+y)}{y \cos x \cdot \cos(x+y)}$$

$$= \lim_{y \to 0} \frac{x[\cos x - \cos(x+y)] + y \cos x}{y \cos x \cdot \cos(x+y)}$$

$$= \lim_{y \to 0} \frac{x \cdot 2 \sin \frac{2x+y}{2} \cdot \sin \frac{y}{2}}{y \cos x \cdot \cos(x+y)}$$

$$= \lim_{y \to 0} \frac{x \cdot 2 \sin \frac{2x+y}{2} \cdot \sin \frac{y}{2}}{y \cos x \cdot \cos(x+y)} + \lim_{y \to 0} \frac{y \cos x}{y \cos x \cdot \cos(x+y)}$$

$$= \lim_{y \to 0} \frac{x \cdot \sin \frac{2x+y}{2}}{\cos x \cdot \cos(x+y)} \cdot \lim_{y \to 0} \frac{\sin y/2}{y/2} + \lim_{y \to 0} \frac{1}{\cos(x+y)}$$

$$= \frac{x \sin x}{\cos x \cdot \cos x} + \frac{1}{\cos x} = \sec x(x \tan x + 1).$$

**S34.** Given, $\lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\left(x - \frac{\pi}{4}\right)}$

Put $x = \frac{\pi}{4} + h$, where $h \to 0$

$$= \lim_{h \to 0} \frac{1 - \tan\left(\frac{\pi}{4} + h\right)}{\left(\frac{\pi}{4} + h - \frac{\pi}{4}\right)} = \lim_{h \to 0} \frac{1 - \tan\left(\frac{\pi}{4} + h\right)}{h}$$

$$= \lim_{h \to 0} \frac{1 - \frac{\left(\tan \frac{\pi}{4} + \tan h\right)}{1 - \tan \frac{\pi}{4} \cdot \tan h}}{h}$$

$$= \lim_{h \to 0} \frac{1 - \left(\dfrac{1 + \tan h}{1 - \tan h}\right)}{h} = \lim_{h \to 0} \frac{1 - \tan h - 1 - \tan h}{h(1 - \tan h)}$$

$$= \lim_{h \to 0} \frac{-2 \tan h}{h(1 - \tan h)} = -2 \lim_{h \to 0} \frac{\tan h}{h} \cdot \frac{1}{1 - \tan h}$$

$$= -2 \times 1 \times \frac{1}{(1-0)} = -2.$$

**S35.** Let, $\displaystyle \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$

Put $\quad x = \dfrac{\pi}{4} - h$

As $\quad x \to \dfrac{\pi}{4} \Rightarrow h \to 0$

$$= \lim_{h \to 0} \frac{\sin\left(\dfrac{\pi}{4} + h\right) - \cos\left(\dfrac{\pi}{4} + h\right)}{\left(\dfrac{\pi}{4} + h - \dfrac{\pi}{4}\right)}$$

$$= \lim_{h \to 0} \frac{\sin\left(\dfrac{\pi}{4} + h\right) - \sin\left(h + \dfrac{3\pi}{4}\right)}{h} \qquad \left\{\because \cos\left(\frac{\pi}{4} + h\right) = \sin\left(\frac{\pi}{4} + h + \frac{\pi}{2}\right) = \sin\left(h + \frac{3\pi}{4}\right)\right\}$$

$$= \lim_{h \to 0} \frac{2\cos\left(h + \dfrac{\pi}{4}\right) \cdot \sin\left(-\dfrac{\pi}{4}\right)}{h} \qquad \left\{\sin C - \sin D = 2\cos\frac{C+D}{2} \cdot \cos\frac{C-D}{2}\right\}$$

$$= \lim_{h \to 0} \frac{(-\sin h)\left[\sin\left(-\dfrac{\pi}{4}\right)\right]}{h}$$

$$= \lim_{h \to 0} 2\sin\frac{\pi}{4}\left(\frac{\sin h}{h}\right)$$

$$= 2\sin\frac{\pi}{4} \times 1 = \sqrt{2}.$$

**S36.** Given, $\displaystyle \lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x}$

Put $\quad x = \left(\dfrac{\pi}{2} - h\right) \qquad \left\{\text{as } x \to \dfrac{\pi}{2},\ h \to 0\right\}$

$$= \lim_{h \to 0} \frac{1 - \sin\left(\frac{\pi}{2} - h\right)}{\cos\left(\frac{\pi}{2} - h\right)}$$

$$= \lim_{h \to 0} \frac{1 - \cos h}{\sin h} = \lim_{h \to 0} \frac{2 \sin^2 \frac{h}{2}}{2 \sin \frac{h}{2} \cdot \cos \frac{h}{2}}$$

$$= \lim_{h \to 0} \tan \frac{h}{2} = 0.$$

**S37.** Given, $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sin^{-1} x}$

Put $x = \sin\theta \Rightarrow \sin^{-1} x = \theta$

$$= \lim_{\theta \to 0} \frac{\sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}}{\theta}$$

$$= \lim_{\theta \to 0} \frac{\left(\sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}\right)}{\theta} \times \frac{\left(\sqrt{1+\sin\theta} + \sqrt{1-\sin\theta}\right)}{\left(\sqrt{1+\sin\theta} + \sqrt{1-\sin\theta}\right)}$$

$$= \lim_{\theta \to 0} \frac{1 + \sin\theta - 1 + \sin\theta}{\theta\left(\sqrt{1+\sin\theta} + \sqrt{1-\sin\theta}\right)}$$

$$= \lim_{\theta \to 0} \frac{2\sin\theta}{\theta\left(\sqrt{1+\sin\theta} + \sqrt{1-\sin\theta}\right)}$$

$$= \lim_{\theta \to 0} \frac{\frac{2\sin\theta}{\theta}}{\sqrt{1+\sin\theta} + \sqrt{1-\sin\theta}} = \frac{2 \times 1}{\sqrt{1+0} + \sqrt{1-0}} = \frac{2}{2} = 1.$$

Date: 12/10/2021

**Q1.** Evaluate: $\lim_{x \to 0} \frac{e^x - \sin x - 1}{x}$.

**Q2.** Evaluate: $\lim_{x \to 0} \frac{e^{4x} - 1}{x}$.

**Q3.** Evaluate: $\lim_{x \to 0} \frac{e^x - 2\sin x - 1}{x}$.

**Q4.** Evaluate: $\lim_{x \to 0} \frac{e^x + \sin x - 1}{x}$.

**Q5.** Evaluate: $\lim_{x \to 0} \frac{e^{4x} - \sin x - 1}{x}$.

**Q6.** Evaluate: $\lim_{x \to 0} \frac{e^x - \tan x - 1}{x}$.

**Q7.** Evaluate: $\lim_{x \to 1} \frac{\log e^x}{x - 1}$.

**Q8.** Evaluate: $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$.

**Q9.** Evaluate: $\lim_{x \to 0} \frac{e^{\sin 2x} - 1}{x}$.

**Q10.** Evaluate: $\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$.

**Q11.** Evaluate: $\lim_{x \to 0} \frac{e^x - x - 1}{x}$.

**Q12.** Evaluate: $\lim_{x \to 0} \frac{e^{2+x} - x^2}{x}$.

**Q13.** Evaluate: $\lim_{x \to 3} \frac{e^x - x^3}{x - 3}$.

**Q14.** Evaluate: $\lim_{x \to 0} \frac{e^{k+x} - e^k}{x}$, $k \in R$.

**Q15.** Evaluate: $\lim_{x \to 0} \frac{e^{1/x}}{e^{1/x+1}}$.

**Q16.** Evaluate: $\lim_{x \to 1} \frac{e^{-x} - e^{-1}}{x - 1}$.

**Q17.** Evaluate: $\lim_{x \to \infty} \left(\frac{x}{x+1}\right)^x$.

**Q18.** For what value of $k$. If $\lim_{x \to 0}(1 + 3x)^{1/x} = k$, is continuous at $x = 0$.

**Q19.** Evaluate: $\lim_{x \to 0} \frac{e^{\sin nx} - 1}{x}$, $(n \in I)$.

**Q20.** Evaluate: $\lim_{x \to 0} \frac{e^{\sin nx} - 1}{nx}$, $n \in I$.

**Q21.** Evaluate: $\lim_{x \to \infty} \left(\frac{x+a}{x+b}\right)^{x+b}$.

**Q22. Evaluate:** $\lim_{x \to 1} x^{1/1-x}$.

**Q23. Evaluate:** $\lim_{x \to 0} \frac{9^x - 7^x}{\sin^{-1} x}$.

**Q24. Evaluate:** $\lim_{x \to 0} \frac{a^x - 1}{b^x - 1}$.

**Q25. Evaluate:** $\lim_{x \to 0} \frac{e^{2x} - 1}{3x}$.

**Q26. Evaluate:** $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$.

**Q27. Evaluate:** $\lim_{x \to 0} \frac{7^x - 5^x}{\sin x}$.

**Q28. Evaluate:** $\lim_{x \to 0} \frac{3^x - 2^x}{\tan x}$.

**Q29. Evaluate:** $\lim_{x \to 0} \frac{e^{ax} - e^{bx}}{x}$.

**Q30. Evaluate:** $\lim_{x \to 0} \frac{x(e^x - 1)}{1 - \cos x}$.

**Q31. Evaluate:** $\lim_{x \to k} \frac{e^x - e^k}{x - k}, \quad (k \in R)$.

**Q32.Evaluate:** $\lim_{x \to 0} \frac{a^x - b^x}{x}$.

**Q33. Evaluate:** $\lim_{x \to \infty} \left( \frac{x}{2 + x} \right)^{2x}$.

**Q34. Evaluate:** $\lim_{x \to \infty} \left( 1 + \frac{4}{x - 1} \right)^{x+3}$.

**Q35. Evaluate:** $\lim_{x \to \infty} \left( \frac{x - 3}{x + 2} \right)^x$.

**Q36. Evaluate:** $\lim_{x \to \infty} \left( \frac{2 + x}{1 + x} \right)^{(2x+1)}$.

**Q37. Evaluate:** $\lim_{x \to 0} (1 - a x)^{1/x}$.

**Q38. Evaluate:** $\lim_{x \to 0} \frac{\log_e (1 + kx)}{x}, \quad k \in R$.

**Q39. Evaluate:** $\lim_{x \to 0} \frac{e^{\alpha x} - e^{\beta x}}{x}$.

**Q40. Evaluate:** $\lim_{x \to 1} \frac{e^{\sqrt{x}} - e^{\sqrt{a}}}{x - a}$.

**Q41. Evaluate:** $\lim_{x \to \frac{\pi}{2}} \frac{e^{\cos x} - 1}{\cos x}$.

**Q42. Evaluate:** $\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{\tan x}$.

**Q43. Evaluate:** $\lim_{x \to 0} (1 - 2x)^{1/x}$.

**Q44. Evaluate:** $\lim_{x \to \infty} \left( \frac{x + 6}{x + 1} \right)^{x+4}$.

**Q45.** Evaluate: $\lim_{x \to \infty} \left\{ \frac{e}{\left(1 + \frac{1}{x}\right)^x} \right\}^x$.

**Q46.** Evaluate: $\lim_{x \to \infty} \left(1 - \frac{4}{x-1}\right)^{3x-1}$.

**Q47.** Evaluate: $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$.

**Q48.** If $a$, $b$, $c$, $d$ are positive real numbers, then find the value of $\lim_{n \to \infty} \left(1 + \frac{1}{a + bn}\right)^{c+dn}$.

**Q49.** Evaluate: $\lim_{x \to 1} (\log_5 5x)^{\log_x 5}$.

**Q50.** Evaluate: $\lim_{x \to 1} (\log_2 2x)^{\log_x 5}$.

**Q51.** Evaluate: $\lim_{x \to \infty} \left(\frac{x^2 - 5x + 3}{x^2 - x + 2}\right)^x$.

**Q52.** Evaluate: $\lim_{x \to \infty} \left(\frac{x^2 - 2x + 1}{x^2 - 4x + 2}\right)^x$.

Date: 12/10/2021

**S1.** $$\lim_{x \to 0} \frac{e^x - \sin x - 1}{x} = \lim_{x \to 0} \left[\frac{e^x - 1}{x} - \frac{\sin x}{x}\right]$$

$$= \lim_{x \to 0} \frac{e^x - 1}{x} - \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 1 - 1 = 0.$$

**S2.** $$\lim_{x \to 0} \frac{e^{4x} - 1}{x} = \lim_{x \to 0} \frac{e^{4x} - 1}{4x} \times 4 \qquad \left[\because \lim_{x \to 0} \frac{e^x - 1}{x} = 1\right]$$

$$= 4 \times 1 = 4$$

**S3.** $$\lim_{x \to 0} \frac{e^x - 2\sin x - 1}{x} = \lim_{x \to 0} \left(\frac{e^x - 1}{x}\right) - 2 \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 1 - 2 = -1.$$

**S4.** $$\lim_{x \to 0} \frac{e^x + \sin x - 1}{x} = \lim_{x \to 0} \frac{e^x - 1}{x} + \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 1 + 1 = 2.$$

**S5.** $$\lim_{x \to 0} \frac{e^{4x} - \sin x - 1}{x} = \lim_{x \to 0} \frac{e^{4x} - 1}{4x} \times 4 - \lim_{x \to 0} \frac{\sin x}{x}$$

$$= 1 \times 4 - 1 = 3.$$

**S6.** $$\lim_{x \to 0} \frac{e^x - \tan x - 1}{x} = \lim_{x \to 0} \frac{e^x - 1}{x} - \lim_{x \to 0} \frac{\tan x}{x}$$

$$= 1 - 1 = 0.$$

**S7.** Put $x = 1 + h$

As $x \to 1, \Rightarrow h \to 0$

$\therefore$ $$\lim_{x \to 1} \frac{\log e^x}{(x - 1)} = \lim_{h \to 0} \frac{\log_e (1 + h)}{h} = 1$$

$\because$ $$\lim_{x \to 0} \frac{\log (1 + x)}{x} = 1.$$

**S8.**

$$\lim_{x\to 0}\frac{e^{\sin x}-1}{x} = \lim_{x\to 0}\frac{e^{\sin x}-1}{\sin x}\times\lim_{x\to 0}\frac{\sin x}{x}$$

$$= 1\times 1 = 1.$$

**S9.**

$$\lim_{x\to 0}\frac{e^{\sin 2x}-1}{x} = \lim_{x\to 0}\frac{e^{\sin 2x}-1}{\sin 2x}\cdot\frac{\sin 2x}{x}$$

$$= \lim_{x\to 0}\frac{e^{\sin 2x}-1}{\sin 2x}\cdot\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2$$

$$= 1\times 2 = 2.$$

**S10.**

$$\lim_{x\to 0}\frac{e^{x}-e^{-x}}{x} = \lim_{x\to 0}\frac{e^{2x}-1}{x}\cdot\lim_{x\to 0}\frac{1}{e^{x}}$$

$$= (2\times 1)\times 1 = 2.$$

**S11.**

$$\lim_{x\to 0}\frac{e^{x}-x-1}{x} = \lim_{x\to 0}\frac{e^{x}-1}{x}-\lim_{x\to 0}\frac{x}{x}$$

$$= 1-1 = 0.$$

**S12.**

$$\lim_{x\to 0}\frac{e^{2+x}-x^{2}}{x} = \lim_{x\to 0}\frac{e^{2}\cdot e^{x}-e^{2}}{x} = e^{2}\cdot\lim_{x\to 0}\frac{e^{x}-1}{x}$$

$$= e^{2}.$$

**S13.** $\lim_{x\to 3}\frac{e^{x}-x^{3}}{x-3}$

Put $x = 3 + h$

$$\Rightarrow\quad \lim_{h\to 0}\frac{e^{3+h}-x^{3}}{3+h-3} = \lim_{h\to 0}\frac{e^{3+h}-e^{3}}{h} = e^{3}\cdot\lim_{h\to 0}\frac{e^{h}-1}{h}$$

$$= e^{3}.$$

**S14.**

$$\lim_{x\to 0}\frac{e^{k+x}-e^{k}}{x} = \lim_{x\to 0}\left(\frac{e^{x}-1}{x}\right)e^{k} = e^{k}. \qquad \left[\because\ \lim_{x\to 0}\frac{e^{x}-1}{x}=1\right]$$

**S15.**

$$\lim_{x\to 0}\frac{e^{1/x}}{e^{(1/x)+1}} = \lim_{x\to 0}e^{\left(\frac{1}{x}-\frac{1}{x}-1\right)} = \lim_{x\to 0}e^{-1} = e^{-1} = \frac{1}{e}.$$

**S16.** $\lim_{x\to 1}\frac{e^{-x}-e^{-1}}{x-1}$ $\qquad\left[\frac{0}{0}\text{ form}\right]$

$$= \lim_{x\to 1}\frac{(-e^{-x})}{x-1} = -e^{-1} = -\frac{1}{e}.$$

**Alternate Method:**

$$\lim_{x\to 1}\frac{e^{-x}-e^{-1}}{x-1} = -\lim_{x\to 1}\frac{e^{-1}-e^{-x}}{x-1}$$

$$= -\left\{\lim_{x\to 1} e^{-x}\right\}.\left\{\lim_{x\to 1}\frac{(e^{x-1}-1)}{(x-1)}\right\}$$

$$= -e^{-1}.1 = -\frac{1}{e}.$$

**S17.**

$$\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x = \lim_{x\to\infty}\left[\frac{1}{\left(1+\frac{1}{x}\right)}\right]^x = \frac{1}{e}.$$

**S18.**

$$\lim_{x\to 0}(1+3x)^{\frac{1}{x}} = e \qquad \left[\because \lim_{x\to 0}(1+px)^{\frac{1}{x}} = e^p\right]$$

$\therefore$ $k = e^3.$

**S19.**

$$\lim_{x\to 0}\frac{e^{\sin nx}-1}{x} = \lim_{x\to 0}\frac{e^{\sin nx}-1}{\sin nx}\cdot\frac{\sin nx}{x}$$

$$= \lim_{x\to 0}\frac{e^{\sin nx}-1}{\sin nx}\cdot\lim_{x\to 0}\frac{\sin nx}{nx}\times n$$

$$= 1\times n = n.$$

**S20.**

$$\lim_{x\to 0}\frac{e^{\sin nx}-1}{nx} = \lim_{x\to 0}\frac{e^{\sin nx}-1}{\sin nx}\cdot\frac{\sin nx}{nx}$$

$$= \lim_{x\to 0}\frac{e^{\sin nx}-1}{\sin nx}\cdot\lim_{x\to 0}\frac{\sin nx}{nx}$$

$$= 1\times 1 = 1.$$

**S21.**

Given,

$$\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{(x+b)} = \lim_{x\to\infty}\left\{\frac{(x+b)+(a-b)}{(x+b)}\right\}^{(x+b)}$$

$$= \lim_{(x+b)\to\infty}\left\{1+\frac{(a-b)}{x+b}\right\}^{(x+b)} = e^{(a-b)} \qquad \left[\because \lim_{t\to\infty}\left(1+\frac{p}{t}\right)^t = e^p\right]$$

**S22.**

$$\lim_{x\to 1}x^{\frac{1}{1-x}} = \lim_{x\to 1}(1+x-1)^{\frac{-1}{x-1}}$$

$$= \lim_{x\to 1}\left[\{1+(x-1)\}^{\frac{1}{x-1}}\right]^{-1} = e^{-1} = \frac{1}{e}. \qquad \left[\because \lim_{x\to 1}(1+t)^{\frac{1}{t}} = e\right]$$

**Alternate Method:**

Let $f(x)$ = x. Then, $\lim_{x \to 1} f(x) = \lim_{x \to 1} x = 1$.

$$\therefore \quad \lim_{x \to 1} \{f(x)\}^{g(x)} = e^{\lim_{x \to 1} g(x)\{f(x) - 1\}}$$

$$\therefore \quad \lim_{x \to 1} \{f(x)\}^{\frac{1}{1-x}} = e^{\lim_{x \to 1} \frac{1}{1-x}.(x-1)} = e^{-1} = \frac{1}{e}.$$

**S23.**
$$\lim_{x \to 0} \frac{9^x - 7^x}{\sin^{-1} x} = \lim_{x \to 0} \frac{(9^x - 1) - (7^x - 1)}{\sin^{-1} x}$$

$$= \lim_{x \to 0} \frac{(9^x - 1)}{x} \cdot \frac{x}{\sin^{-1} x} - \lim_{x \to 0} \frac{(7^x - 1)}{x} \cdot \frac{x}{\sin^{-1} x}$$

$$= \log 9 - \log 7 = \left(\log 9/7\right).$$

**S24.**
$$\lim_{x \to 0} \frac{a^x - 1}{b^x - 1} = \lim_{x \to 0} \frac{\frac{a^x - 1}{x}}{\frac{b^x - 1}{x}}$$

$$= \frac{\log_e a}{\log_e b} = \log_b a.$$

**S25.**
$$\lim_{x \to 0} \frac{e^{2x} - 1}{3x} = \lim_{x \to 0} \frac{e^{2x} - 1}{2x} \times \frac{2}{3}$$

$$= \frac{2}{3} \lim_{x \to 0} \frac{e^{2x} - 1}{2x} = \frac{2}{3}.$$

**S26.**
$$\lim_{x \to 0} \frac{e^x + \frac{1}{e^x} - 2 \cdot e^x \cdot \frac{1}{x}}{x^2} = \lim_{x \to 0} \frac{(e^x - 1)^2}{x^2}$$

$$= \lim_{x \to 0} \left(\frac{e^x - 1}{x}\right)^2 = 1^2 = 1.$$

**S27.**
$$\lim_{x \to 0} \frac{7^x - 5^x}{\sin x} = \lim_{x \to 0} \frac{(7^x - 1) - (5^x - 1)}{\sin x}$$

$$= \lim_{x \to 0} \left(\frac{7^x - 1}{x}\right) \cdot \frac{x}{\sin x} - \lim_{x \to 0} \left(\frac{5^x - 1}{x}\right) \cdot \frac{x}{\sin x}$$

$$= \log 7 - \log 5 = \log \frac{7}{5}.$$

**S28.**

$$\lim_{x\to 0}\frac{3^x-2^x}{\tan x} = \lim_{x\to 0}\frac{(3^x-1)-(2^x-1)}{\tan x}$$

$$= \lim_{x\to 0}\frac{3^x-1}{x}\times\frac{x}{\tan x} - \lim_{x\to 0}\frac{2^x-1}{x}\cdot\frac{x}{\tan x}$$

$$= \log 3 - \log 2 = \log\frac{3}{2}.$$

**S29.**

$$\lim_{x\to 0}\frac{(e^{ax}-e^{bx})}{x} = \lim_{x\to 0}\frac{(e^{ax}-1)-(e^{bx}-1)}{x}$$

$$= \lim_{x\to 0}\frac{e^{ax}-1}{ax}\times a - \lim_{x\to 0}\frac{e^{bx}-1}{bx}\times b$$

$$= (a-b).$$

**S30.**

$$\lim_{x\to 0}\frac{x(e^x-1)}{2\sin^2\frac{x}{2}} = \frac{1}{2}\cdot\lim_{x\to 0}\frac{x(e^x-1)}{\sin\frac{x}{2}\cdot\sin\frac{x}{2}}$$

$$= \frac{1}{2}\cdot\lim_{x\to 0}\frac{e^x-1}{x}\cdot\lim_{x\to 0}\frac{\left(\frac{x}{2}\right)^2\times 4}{\sin^2\frac{x}{2}}$$

$$= \frac{1}{2}\times 1\times 4 = 2.$$

**S31.** Given, $\lim_{x\to k}\frac{e^x-e^k}{x-k}$

Put $x = k + h$

$$= \lim_{h\to 0}\frac{e^{k+h}-e^k}{k+h-k} = \lim_{h\to 0}\frac{e^{k+h}-e^k}{h}$$

$$= e^k\cdot\lim_{h\to 0}\frac{e^h-1}{h}$$

$$= e^k.$$

**S32.**

$$\lim_{x\to 0}\frac{a^x-b^x}{x} = \lim_{x\to 0}\frac{(a^x-1)-(b^x-1)}{x}$$

$$= \lim_{x\to 0}\left(\frac{a^x-1}{x}-\frac{b^x-1}{x}\right)$$

$$= \lim_{x \to 0} \frac{a^x - 1}{x} - \lim_{x \to 0} \frac{b^x - 1}{x}$$

$$= \log a - \log b = \log \frac{a}{b}.$$

**S33.** Given,

$$\lim_{x \to \infty} \left(\frac{x}{2+x}\right)^{2x} = \lim_{x \to \infty} \left(1 + \frac{-2}{x+2}\right)^{2x} = \lim_{x \to \infty} \left[\left(1 + \frac{-2}{x+2}\right)^{x+2}\right]^{\frac{2x}{x+2}}$$

$$= (e^{-2})^{\lim_{x \to \infty}\left(\frac{1}{1+\frac{2}{x}}\right)} \qquad \left[\because \lim_{x \to \infty}\left(1+\frac{p}{x}\right)^x = e^p\right]$$

$$= (e^{-2})^2 = e^{-4}.$$

**S34.**

$$\lim_{x \to \infty} \left(1 + \frac{4}{x-1}\right)^{(x+3)} = \lim_{x \to \infty} \left[\left\{1 + \frac{4}{x-1}\right\}^{(x-1)}\right]^{\left(\frac{x+3}{x-1}\right)}$$

$$= (e^4)^{\lim_{x \to \infty}\left(\frac{x+3}{x-1}\right)} \qquad \left[\because \lim_{t \to \infty}\left(1+\frac{p}{t}\right)^t = e^p\right]$$

$$= (e^4)^{\lim_{t \to \infty}\left(\frac{1+\frac{3}{x}}{1-\frac{1}{x}}\right)} = (e^4)^{\left(\frac{1+0}{1-0}\right)} = (e^4)^1 = e^4.$$

**S35.**

$$\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x = \lim_{x \to \infty} \left(1 + \frac{-5}{x+2}\right)^x$$

$$= \lim_{x \to \infty} \left[\left(1 + \frac{-5}{x+2}\right)^{x+2}\right]^{\frac{x}{x+2}}$$

$$= (e^{-5})^{\lim_{x \to \infty}\left(\frac{1}{1+\frac{2x}{x}}\right)} \qquad \left[\lim_{t \to \infty}\left(1+\frac{p}{t}\right)^t = e^p\right]$$

$$= (e^{-5})^{\left(\frac{1}{1+0}\right)} = (e^{-5})^1 = e^{-5}.$$

**S36.**

$$\lim_{x \to \infty} \left(\frac{2+x}{1+x}\right)^{(2x+1)} = \lim_{x \to \infty} \left[\left(1 + \frac{1}{1+x}\right)^{x+1}\right]^{\left(\frac{2x+1}{x+1}\right)}$$

$$= e^{\lim_{x \to \infty}\left(\frac{2x+1}{1+x}\right)} = e^{\lim_{x \to \infty}\left(\frac{2+\frac{1}{x}}{1+\frac{1}{x}}\right)} = e^{\frac{2+0}{1+0}} = e^2.$$

**S37.** Let

$$L = \lim_{x \to 0} (1 - ax)^{1/x}.$$

Then, $$\log L = \lim_{x \to 0} \frac{\log(1 - ax)}{x} \qquad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to 0} \left\{ \frac{\left(\frac{-a}{1 - ax}\right)}{1} \right\} = -a$$

$\Rightarrow$ $$L = e^{-a}.$$

**S38.** $$\lim_{x \to 0} \frac{\log_e(1 + kx)}{x} = 1 \qquad \left[\lim_{x \to 0} \frac{\log_e(1 + kx)}{kx} = 1\right]$$

$$= \lim_{x \to 0} \frac{\log_e(1 + kx)}{kx} \times k$$

$$= 1 \times k = k.$$

**S39.** $$\lim_{x \to 0} \frac{e^{\alpha x} - e^{\beta x}}{x}$$

$$= \lim_{x \to 0} \frac{\alpha e^{\alpha x} - \beta e^{\beta x}}{1} = \alpha e^0 - \beta e^0 = \alpha - \beta.$$

Another Method: $$= \lim_{x \to 0} \frac{e^{\alpha x} - e^{\beta x}}{1} = \lim_{x \to 0} \frac{(e^{\alpha x} - 1) - (e^{\beta x} - 1)}{x}$$

$$= \lim_{x \to 0} \frac{(e^{\alpha x} - 1)}{x} - \lim_{x \to 0} \frac{(e^{\beta x} - 1)}{x}$$

$$= \alpha \left( \lim_{x \to 0} \frac{e^{\alpha x} - 1}{\alpha x} \right) - \beta \left( \lim_{x \to 0} \frac{(e^{\beta x} - 1)}{\beta x} \right)$$

$$= \alpha .1 - \beta .1 = \alpha - \beta.$$

**S40.** Given, $$\lim_{x \to a} \frac{e^{\sqrt{x}} - e^{\sqrt{a}}}{x - a} \qquad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x \to a} \frac{e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}{1} = \frac{e^{\sqrt{a}}}{2\sqrt{a}}.$$

(Alternate Method) $$\lim_{x \to a} \frac{e^{\sqrt{x}} - e^{\sqrt{a}}}{x - a} = \lim_{x \to a} \frac{e^{\sqrt{a}} \cdot (e^{\sqrt{x} - \sqrt{a}} - 1)}{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}$$

$$= e^{\sqrt{a}} \left\{ \lim_{x \to a} \frac{e^{\sqrt{x} - \sqrt{a}} - 1}{\sqrt{x} - \sqrt{a}} \right\} \cdot \left\{ \lim_{x \to a} \frac{1}{\sqrt{x} - \sqrt{a}} \right\}$$

$$= e^{\sqrt{a}} \times 1 \times \frac{1}{2\sqrt{a}} = \frac{e^{\sqrt{a}}}{2\sqrt{a}}.$$

**S41.** $\lim\limits_{x \to \frac{\pi}{2}} \frac{e^{\cos x} - 1}{\cos x}$

Put $x = \frac{\pi}{2} + h$ as $x \to \frac{\pi}{2}, h \to 0$

$$\lim_{h \to 0} \frac{e^{\cos\left(\frac{\pi}{2}+h\right)} - 1}{\cos\left(\frac{\pi}{2}+h\right)} = \lim_{h \to 0} \frac{e^{-\sin x} - 1}{-\sin x} = 1.$$

**S42.**
$$\lim_{x \to 0} \frac{10^x - 2^x - 5^x + 1}{\tan x} = \lim_{x \to 0} \frac{2^x(5^x - 1) - 1(5^x - 1)}{\tan x}$$

$$= \lim_{x \to 0} \frac{(2^x - 1)}{\tan x} - \lim_{x \to 0} \frac{(5^x - 1)}{\tan x}$$

$$= \lim_{x \to 0} \frac{2^x - 1}{x} \cdot \frac{x}{\tan x} - \lim_{x \to 0} \frac{5^x - 1}{x} \cdot \frac{x}{\tan x}$$

$$= \log 2 - \log 5 = \left(\log 2/5\right).$$

**S43.** Let
$$L = \lim_{x \to 0} (1 - 2x)^{1/x}$$

Then,
$$\log L = \lim_{x \to 0} \frac{\log(1 - 2x)}{x} \qquad \left[\frac{0}{0}\text{ from}\right]$$

$$= \lim_{x \to 0} \frac{\left(\frac{-2}{1-2x}\right)}{1} = \frac{-2}{1-0} = -2.$$

∴
$$L = e^{-2}.$$

**Alternate Method:**

$$\lim_{x \to 0} (1 - 2x)^{1/x} = \lim_{x \to 0} \left[\{1 + (-2x)\}^{\left(\frac{1}{-2x}\right)}\right]^{-2}$$

$$= e^{-2}.$$

**S44.**
$$\lim_{x \to 0} \left(\frac{x+6}{x+1}\right)^{x+4} = \lim_{x \to 0} \left\{\left(1 + \frac{5}{x+1}\right)^{\frac{x+1}{5}}\right\}^{\frac{5(x+4)}{x+1}}$$

$$= e^{\lim_{t\to\infty} \frac{5(x+4)}{x+1}} \qquad \left[\because \lim_{t\to 0}(1+t)^{\frac{1}{t}} = 0 \text{ and } \lim_{t\to\infty}\frac{5}{x+1} = 0\right]$$

$$= e^{\lim_{t\to\infty}\left\{\frac{5\left(1+\frac{4}{x}\right)}{\left(1+\frac{1}{x}\right)}\right\}}$$

$$= e^{\frac{5(1+0)}{(1+0)}} = e^5.$$

**S45.** Let

$$L = \lim_{x\to\infty}\left\{\frac{e}{\left(1+\frac{1}{x}\right)^x}\right\}^x$$

$$\Rightarrow \qquad \log L = \lim_{x\to\infty} x\log\left\{\frac{e}{\left(1+\frac{1}{x}\right)^x}\right\}$$

$$\lim_{x\to\infty}\left\{x\log e - x^2\log\left(1+\frac{1}{x}\right)\right\} = \lim_{x\to\infty}\left\{x - x^2\log\left(1+\frac{1}{x}\right)\right\}$$

Put $t = \frac{1}{x}$.

Then, as $x \to \infty$, $t \to 0$.

$$\therefore \qquad \log L = \lim_{t\to 0}\frac{1}{t} - \frac{1}{t^2}\log(1+t)$$

$$= \lim_{t\to 0}\frac{1}{t^2}[t - \log(1+t)] = \lim_{t\to 0}\left\{\frac{t - \left(t - \frac{t^2}{2} + \frac{t^3}{3} - \ldots\ldots\right)}{t^2}\right\}$$

$$= \lim_{t\to 0}\frac{\left(\frac{t^2}{2} - \frac{t^3}{3} + \ldots..\right)}{t^2}$$

$$= \lim_{t\to 0}\left(\frac{1}{2} + \text{terms containing powers of } t\right) = \frac{1}{2}.$$

$$\therefore \qquad L = e^{1/2}.$$

**S46.** Let

$$L = \lim_{t\to 0}\left(1 - \frac{4}{x-1}\right)^{3x-1}$$

$$= \lim_{t\to\infty}\left[\left(1 - \frac{4}{x-1}\right)^{(x-1)}\right]^{\frac{3x-1}{x-1}}$$

$$= (e^{-4})^{\lim_{t\to\infty}\left(\frac{3x-1}{x-1}\right)} \qquad \left[\because \lim_{t\to\infty}\left(1+\frac{p}{t}\right)^t = e^p\right]$$

$$= (e^{-4})^{\lim_{t\to\infty}\left(\frac{3-\frac{1}{x}}{1-\frac{1}{x}}\right)} = (e^{-4})^{\left(\frac{3-0}{1-0}\right)} = (e^{-4})^3 = e^{-12}.$$

**S47.** Let
$$L = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

$$\Rightarrow \qquad \log L = \lim_{x\to\infty} x\log\left(1+\frac{1}{x}\right)$$

$$= \lim_{x\to\infty} x\left\{\frac{1}{x}-\frac{1}{2}\left(\frac{1}{x}\right)^2+\frac{1}{3}\left(\frac{1}{x}\right)^3-......\right\}$$

$$= \lim_{x\to\infty}\left\{1-\frac{1}{2}\left(\frac{1}{x}\right)+\frac{1}{3}\left(\frac{1}{x^2}\right)-......\right\} = 1-0+0-.....=1$$

$$\Rightarrow \qquad L = e^1 = e.$$

**S48.**
$$\lim_{n\to\infty}\left(1+\frac{1}{a+bn}\right)^{c+dn} = \lim_{n\to\infty}\left[\left(1+\frac{1}{a+bn}\right)^{(a+bn)}\right]^{\frac{c+dn}{a+bn}}$$

$$= e^{\lim_{n\to\infty}\frac{c+dn}{a+bn}} \qquad \left[\because \lim_{n\to\infty}\left(1+\frac{1}{t}\right)^t = e\right]$$

$$= e^{\lim_{t\to\infty}\left\{\frac{\left(\frac{c}{n}+d\right)}{\left(\frac{a}{n}+b\right)}\right\}}$$

$$= e^{\left(\frac{0+d}{0+b}\right)} = e^{d/b.}$$

**S49.** Let
$$L = \lim_{x\to 1}(\log_5 5x)^{\log_x 5}$$

$$= \lim_{x\to 1}(\log_5 5+\log_5 x)^{\frac{1}{\log_5 x}}$$

Put $\log_5 x = t$. Then, as $x \to 1$, $t \to 0$.

$$\therefore \qquad L = \lim_{t\to 0}(1+t)^{\frac{1}{t}} = e.$$

**S50.** Let

$$L = \lim_{x \to 1} (\log_2 2x)^{\log_x 5}$$

$$= \lim_{x \to 1} (\log_2 2 + \log_2 x)^{\left(\frac{\log 5}{\log 2} \cdot \frac{\log 2}{\log x}\right)}$$

$$= \lim_{x \to 1} \left[ (1 + \log_2 x)^{\frac{1}{\log_2 x}} \right]^{\log_2 5}$$

Put $\log_2 x = t$. Then, as $x \to 1$, $t \to 0$.

$\therefore$ $$L = \lim_{t \to 0} \left[ (1+t)^{\frac{1}{t}} \right]^{\log_2 5} = e^{\log_2 5}. \qquad \left[ \because \lim_{t \to 0} (1+t)^{\frac{1}{t}} = e \right]$$

**S51.** Let $$L = \lim_{x \to \infty} \left( \frac{x^2 + 5x + 3}{x^2 + x + 2} \right)^x = \lim_{x \to \infty} \left( 1 + \frac{4x+1}{x^2 + x + 2} \right)^x$$

$$= \lim_{x \to \infty} \left[ \left( 1 + \frac{4x+1}{x^2+x+2} \right)^{\frac{x^2(4x+1)}{x^2+x+2}} \right]^{\frac{x(4x+1)}{x^2+x+2}}$$

$$= \left\{ \lim_{x \to \infty} \left( 1 + \frac{4x+1}{x^2+x+2} \right)^{\frac{(x^2+x+2)}{4x+1}} \right\}^{\lim_{x \to \infty} \frac{(4x^2+x)}{x^2+x+2}}$$

Now, $$\lim_{x \to \infty} \frac{4x+1}{x^2+x+2} = \lim_{x \to \infty} \frac{4 + \frac{1}{x}}{x + 1 + \frac{2}{x}} = \frac{4}{\infty} = 0.$$

we put $t = \dfrac{4x+1}{x^2+x+2}$. Then, $t \to 0$ as $x \to \infty$.

$$\therefore \quad \lim_{x \to \infty} \left( 1 + \frac{4x+1}{x^2+x+2} \right)^{\frac{x^2+x+2}{4x+1}} = \lim_{x \to 0} (1+t)^{1/t} = e.$$

Also $$\lim_{x \to \infty} \left( \frac{4x^2+x}{x^2+x+2} \right) = \lim_{x \to \infty} \frac{4 + \frac{1}{x}}{1 + \frac{1}{x} + \frac{2}{x^2}} = \frac{4+0}{1+0+0} = 4.$$

Hence, $L = e^4$.

**S52.** Let

$$L = \lim_{t\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^x = \lim_{t\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^x$$

$$= \lim_{t\to\infty}\left[\left(1+\frac{2x-1}{x^2-4x+2}\right)^{\frac{x^2-4x+2}{2x-1}}\right]^{\frac{x(2x\ -1)}{(x^2-4x+2)}}$$

$$= \left[\lim_{t\to\infty}\left(1+\frac{2x-1}{x^2-4x+2}\right)^{\frac{x^2-4x+2}{2x-1}}\right]^{\frac{2x^2-x}{(x^2-4x+2)}}$$

Now, $$\lim_{t\to\infty}\frac{2x-1}{x^2-4x+2} = \lim_{t\to\infty}\frac{2-\frac{1}{x}}{x-4+\frac{2}{x}} = \frac{2}{\infty} = 0.$$

we put $$t = \frac{2x-1}{x^2-4x+2}. \text{ Then, } T\to 0 \text{ as } x\to\infty.$$

$$\therefore \quad \lim_{x\to\infty}\left(1+\frac{2x+1}{x^2-4x+2}\right)^{\left(\frac{x^2-4x+2}{2x-1}\right)} = \lim_{x\to\infty}(1+t)^{\frac{1}{t}} = e.$$

Also, $$\lim_{x\to\infty}\left(\frac{2x^2-1}{x^2-4x+2}\right) = \lim_{x\to\infty}\frac{2-\frac{1}{x}}{1-\frac{4}{x}+\frac{2}{x^2}} = \frac{2-0}{1-0+0} = 2.$$

Hence $L = e^2$.

Date: 12/10/2021

**Q1.** Find the derivative of $f(x) = 10x$.

**Q2.** Find the derivative of $f(x) = x^2$.

**Q3.** Evaluate: $\lim_{x \to 1} \frac{x^{15} - 1}{x^{10} - 1}$.

**Q4.** Find the derivative of $x$ at $x = 1$.

**Q5.** Find the derivative of the constant function $f(x) = a$ for a fixed real number $a$.

**Q6.** Find the derivative of $-x$ from first principle

**Q7.** Find the derivative of $\sin x$ at $x = 0$.

**Q8.** Find the derivative of $f(x) = 3$ at $x = 0$ and $x = 3$.

**Q9.** Differentiate from first principles $f(x) = \frac{1}{x}, x > 0$.

**Q10.** Find the derivative at $x = 2$ of the function $f(x) = 3x$.

**Q11.** Find the value of $n$. If $\lim_{x \to 3} \frac{x^n - 3^n}{x - 3} = 108$.

**Q12.** Find all possible values of $n$ if $\lim_{x \to 2} \frac{x^n - 2^n}{(x - 2)} = 80, n \in N$.

**Q13.** Find the value of $a$, ($a \in R$). If $\lim_{x \to a} \frac{x^9 - a^9}{(x - a)} = 9$.

**Q14.** Find the derivative of $(x^3 - 27)$ from first principle.

**Q15.** Find the value of $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n}$.

**Q16.** Find the derivative of $x^2 - 2$ at $x = 10$ from first principle.

**Q17.** Find the derivative of $ax + b$ from first principle.

**Q18.** Evaluate: $\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{3/7} - a^{3/7}}$.

**Q19.** Find the value of $\lambda$ if

$$\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to \lambda} \frac{x^3 - \lambda^3}{x^2 - \lambda^2}$$

**Q20.** Find the derivative of $\frac{ax + b}{cx + d}$ from first principle.

**Q21.** Find the derivative of $\frac{1}{\sqrt{x}}$ from first principle.

**Q22.** Find the derivative of $\sqrt{x}$ from first principle.

**Q23.** Find the derivative of $ax^2 + bx + c$ from first principle.

**Q24.** Find the derivative of $x^n$ from first principle.

**Q25.** Find the derivative of $x \cos x$ function from first principle.

**Q26.** Find the derivative of $x \sin x$ function from first principle.

**Q27.** Find the derivative of $\cot x$ function from first principle.

**Q28.** Find the derivative of $\operatorname{cosec} x$ function from first principle.

**Q29.** Find the derivative of $\sec x$ function from first principle.

**Q30.** Find the derivative of $\tan x$ function from first principle.

**Q31.** Find the derivative of $\cos x$ from first principle.

**Q32.** Find the derivative of $\sin x$ from first principle.

**Q33.** Differentiate from first principles $f(x) = (x^2 + 1)(x - 5)$

**Q34.** Differentiate from first principles $f(x) = (x + 1)(x + 2)(x + 3)$.

**Q35.** Differentiate from first principles $f(x) = (x + 1)(x + 2)$.

**Q36.** Differentiate from first principles $f(x) = x^4 + 1$.

**Q37.** Differentiate from first principles $f(x) = x^3$.

**Q38.** Find the derivative of $\frac{1}{x^2}$ from first principle.

[redacted]

**Q41.** Find the derivative of $x^2 - 2$ at $x = 10$.

**Q42.** Find the derivative of the function $f(x) = 2x^2 + 3x - 5$ at $x = -1$, Also prove that $f'(0) + 3f'(-1) = 0$.

[redacted]

**Q46.** Differentiate from first principles $f(x) = (ax + b)^n$.

**Q47.** Find the derivative of the following functions from first principle: $\frac{x+1}{x-1}$.

**Q48.** Find the derivative of $99x$ at $x = 100$.

**Q49.** If $f(x) = x^2 - 19x + 18$, then find $f'(2)$, $f'(1)$ and $f'(10)$ using limit process only once.

**Q50.** Find the derivative of $x + \frac{1}{x}$ w.r.t. $x$ from the first principles.

**Q51.** Find the derivative of the function $3x^3 + 2x^2 + 3x - 6$ at $x = -1$. Also, prove that $8f'(0) - 3f'(-1) = 0$.

**Q52.** Differentiate from first principles $f(x) = \dfrac{ax^2 + bx + c}{\sqrt{x}}$.

**Q53.** Differentiate from first principles $f(x) = \sqrt{x + 1}$, when $x > -1$.

**Q54.** Differentiate from first principles $f(x) = \dfrac{2 - x}{4 + 3x}$, $x > -\dfrac{4}{3}$.

**Q55.** Find $a$ if $f'(a) = 0$, where $f(x) = x^3 - 3x^2 + 3x - 1$

**Q56.** Find the derivative of $\cos\left(x - \dfrac{\pi}{8}\right)$ from first principles.

**Q57.** Differentiate $\sec\left(\dfrac{x}{2} - 1\right)$ w.r.t. x from the first principle.

**Q58.** Differentiate $\cos(x^2 + 1)$ w.r.t. x from the first principles.

**Q59.** Differentiate $\dfrac{\sin x}{x}$ from first principles.

**Q60.** Differentiate $\dfrac{\sin x}{x - 6}$ w.r.t. x from the first principle.

Date: 12/10/2021

**S1.** Since,
$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{10(x+h)-10(x)}{h}$$
$$= \lim_{h\to 0}\frac{10h}{h} = \lim_{h\to 0}(10) = 10.$$

**S2.** We have,
$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{(x+h)^2-(x)^2}{h}$$
$$= \lim_{h\to 0}(h+2x) = 2x.$$

**S3.**
$$\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1} = \lim_{x\to 1}\frac{\left(\dfrac{x^{15}-1}{x-1}\right)}{\left(\dfrac{x^{10}-1}{x-1}\right)}$$
$\Rightarrow$
$$\frac{15}{10} = \frac{3}{2}.$$

**S4.** We have,
$$f(x) = x$$
$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$
$$= \lim_{h\to 0}\frac{(1+h)-1}{h} = \lim_{h\to 1} 1 = 1.$$

**S5.** We have,
$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{a-a}{h} = \lim_{h\to 0}\frac{0}{h} = 0 \text{ as } h\neq 0.$$

**S6.** Here,
$$f(x) = -x$$
$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$ [Formula]

When, $f(x) = -x$

$$f'(x) = \lim_{h \to 0} \frac{f(-x+h) - f(-x)}{h}$$ **Ans.**

**S7.** Let $f(x) = \sin x$

Then $$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(0+h) - \sin(0)}{h} = \lim_{h \to 0} \frac{\sin h}{h} = 1.$$

**S8.** Since, the derivative measures the change in function, it is clear that the derivative of the constant function must be Zero at every point. This is indeed, supported by the following computation.

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{3-3}{h} = \lim_{h \to 0} \frac{0}{h} = 0$$

Similarly, $$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{3-3}{h} = 0$$

**S9.** Let $f(x) = \frac{1}{x}$

$$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x} = \frac{x-(x+h)}{x(x+h)} = \frac{x-x-h}{x(x+h)} = \frac{h}{x(x+h)}$$

Taking limit on both sides as $h \to 0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{1}{x(x+h)}, \quad f'(x) = \frac{1}{x^2}.$$

**S10.** We have, $$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{3(2+h) - 3(2)}{h}$$

$$= \lim_{h \to 0} \frac{6+3h-6}{h} = \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3 = 3$$

The derivative of the function $3x$ at $x = 2$ is 3.

**S11.** We have,

$$\lim_{x \to 3} \frac{x^n - 3^n}{x-3} = n \cdot 3^{n-1}$$

$\because$ $$\lim_{x \to a} \frac{x^n - a^n}{(x-a)} = n \cdot a^{n-1}$$

$\because$ $n \cdot 3^{n-1} = 108$

$\Rightarrow$ $n \cdot 3^{n-1} = 4 \cdot 3^{4-1}$

$\Rightarrow$ $n = 4.$

**S12.** We have,

$$\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$$

$\Rightarrow$ $n \,.\, 2^{n-1} = 80$ $\left\{ \lim_{x \to a} \frac{x^n - a^n}{(x - a)} = na^{n-1} \right\}$

$\Rightarrow$ $n \,.\, 2^{n-1} = 5 \,.\, 2^{5-1}$

$\Rightarrow$ $n = 5.$

**S13.** We have,

$$\lim_{x \to a} \frac{x^9 - a^9}{(x - a)} = 9 \,.\, a^8 \qquad \left\{ \lim_{x \to a} \frac{x^n - a^n}{(x - a)} = n \,.\, a^{n-1} \right\}$$

$\because$ $9 \,.\, a^8 = 9$

$\Rightarrow$ $a^8 = 1$

$\Rightarrow$ $a = \pm 1.$

**S14.** We have, $f(x) = x^3 - 27$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)^3 - 27] - [x^3 - 27]}{h}$$

$$= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 27 - x^3 + 27}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \to 0} (3x^2 + 3xh + h^2) = 3x^2.$$ **Ans.**

**S15.** $\because$

$$\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n} = \lim_{x \to a} \left\{ \frac{x^m - a^m}{(x - a)} \cdot \frac{(x - a)}{x^n - a^n} \right\}$$

$$= \lim_{x \to a} \left\{ \frac{x^m - a^m}{(x - a)} \right\} \Big/ \lim_{x \to a} \left\{ \frac{x^n - a^n}{(x - a)} \right\}$$

$$= \frac{ma^{m-1}}{na^{n-1}} = \frac{m}{n} \cdot a^{m-n}.$$

**S16.** We have, $f(x) = x^2 - 2$

$$f'(10) = \lim_{h \to 0} \frac{f(10+h) - f(10)}{h} = \lim_{h \to 0} \frac{[(10+h)^2 - 2] - [(10)^2 - 2]}{h}$$

$$= \lim_{h \to 0} \frac{100 + 20h + h^2 - 2 - 100 + 2}{h} = \lim_{h \to 0} \frac{h^2 + 20h}{h} = \lim_{h \to 0} (h + 20)$$

$= 20,$ **Ans.**

**S17.** Given, $y = ax + b$.

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $y + \Delta y = a(x + \Delta x) + b$

$\Rightarrow$ $\Delta y = a(x + \Delta x) + b - (ax + b)$

$= a\Delta x \Rightarrow \dfrac{\Delta y}{\Delta x} = a$

$\therefore$ $$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (a) = a$$

**S18.**

$$\lim_{x \to a} \frac{x^{5/7} - a^{5/7}}{x^{3/7} - a^{3/7}} = \lim_{x \to a} \frac{\dfrac{x^{5/7} - a^{5/7}}{(x-a)}}{\dfrac{x^{3/7} - a^{3/7}}{(x-a)}} = \lim_{x \to a} \frac{\frac{5}{7} . a^{(5/7-1)}}{\frac{3}{7} . a^{(3/7-1)}}$$

$$= \frac{5 \cdot a^{-2/7}}{3 \cdot a^{-4/7}} = \frac{5}{3} . a^{2/7}.$$

**S19.**

$$\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to \lambda} \frac{x^3 - \lambda^3}{x^2 - \lambda^2}$$

$\because$ $$\lim_{x \to \lambda} \frac{x^3 - \lambda^3}{x^2 - \lambda^2} \times \frac{(x-\lambda)}{(x-\lambda)} = 3\lambda^{(3-1)} \times \frac{1}{2\lambda^{2-1}} \Rightarrow \frac{3}{2}\lambda$$

$\because$ $$\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = 4.(1)^{4-1} = 4$$

$\therefore$ $$\frac{3}{2}\lambda = 4 \Rightarrow \lambda = \frac{8}{3}.$$

**S20.** Given, $y = \dfrac{ax+b}{cx+d}$.

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $$y + \Delta y = \frac{a(x+\Delta x)+b}{c(x+\Delta x)+d}$$

$\Rightarrow$ $$\Delta y = \frac{a(x+\Delta x)+b}{c(x+\Delta x)+d} - \frac{ax+b}{cx+d}$$

$$= \frac{[a(x+\Delta x)+b](cx+d) - [c(x+\Delta x)+d](ax+b)}{[c(x+\Delta x)+d](cx+d)}$$

$$= \frac{(ad-bc)\Delta x}{[c(x+\Delta x)+d](cx+d)}$$

$$\Rightarrow \quad \frac{\Delta y}{\Delta x} = \frac{ad - bc}{[c(x + \Delta x) + d](cx + d)}$$

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{ad - bc}{[c(x + \Delta x) + d](cx + d)} = \frac{ad - bc}{(cx + d)^2}.$$

**S21.** Given,
$$y = \frac{1}{\sqrt{x}}.$$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then,
$$y + \Delta y = \frac{1}{\sqrt{x + \Delta x}}$$

$$\Rightarrow \quad \Delta y = \frac{1}{\sqrt{x + \Delta x}} - \frac{1}{\sqrt{x}}$$

$$= \frac{\sqrt{x} - \sqrt{x + \Delta x}}{\sqrt{x}\sqrt{x + \Delta x}} = \frac{\sqrt{x} - \sqrt{x + \Delta x}}{\sqrt{x}\sqrt{x + \Delta x}} \cdot \frac{\sqrt{x} + \sqrt{x + \Delta x}}{\sqrt{x} + \sqrt{x + \Delta x}}$$

$$= \frac{x - (x + \Delta x)}{\sqrt{x}\sqrt{x + \Delta x}(\sqrt{x} + \sqrt{x + \Delta x})}$$

$$\Rightarrow \quad \frac{\Delta y}{\Delta x} = \frac{-1}{\sqrt{x}\sqrt{x + \Delta x}(\sqrt{x} + \sqrt{x + \Delta x})}$$

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{\sqrt{x}\sqrt{x + \Delta x}(\sqrt{x} + \sqrt{x + \Delta x})}$$

$$= \frac{-1}{2x\sqrt{x}}.$$

**S22.** Given,
$$y = \sqrt{x}.$$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then,
$$y + \Delta y = \sqrt{x + \Delta x}$$

$$\Rightarrow \quad \Delta y = (\sqrt{x + \Delta x} - \sqrt{x}) \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$= \frac{x + \Delta x - x}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$\Rightarrow \quad \frac{\Delta y}{\Delta x} = \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

Given, $y = ax^2 + bx + c$.

Let $\Delta x$ be a small change in $x$ and $\Delta y$ be the corresponding change in $y$.

Then $y + \Delta y = a(x + \Delta x)^2 + b(x + \Delta x) + c$

$$\Rightarrow \quad \Delta y = a(x + \Delta x)^2 + b(x + \Delta x) - (ax^2 + bx)$$

$$= 2xa\Delta x + \Delta x^2 + b\Delta x$$

$$= \Delta x(2ax + \Delta x + b)$$

$$\frac{\Delta y}{\Delta x} = 2ax + \Delta x + b$$

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x \to 0} (2ax + \Delta x + b)$$

$$= 2ax + b$$

**S24.** Given, $y = x^n$.

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $y + \Delta y = (x + \Delta x)^n$. $\quad \because y = x^n$

$$\Rightarrow \quad \Delta y = (x + \Delta x)^n - x^n$$

$$= x^n\left[\left(1 + \frac{\Delta x}{x}\right)^n - 1\right]$$

$$= x^n\left[1 + n\frac{\Delta x}{x} + \frac{n(n-1)}{2!}\left(\frac{\Delta x}{x}\right)^2 + \ldots - 1\right]$$

$$= x^n \Delta x\left[\frac{n}{x} + \frac{n(n-1)}{2!}\frac{\Delta x}{x^2} + \ldots\right]$$

$$\Rightarrow \quad \frac{\Delta y}{\Delta x} = x^n\left[\frac{n}{x} + \frac{n(n-1)}{2!}\frac{\Delta x}{x^2} + \ldots\right]$$

$\therefore \qquad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = x^n\left(\frac{n}{x}\right) = nx^{n-1}$

**S25.** Given, $\qquad y = x \cos x.$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $\qquad y + \Delta y = (x + \Delta x) \cos (x + \Delta x)$

$$\Rightarrow \qquad \Delta y = (x + \Delta x) \cos (x + \Delta x) - x \cos x$$

$$= x[\cos (x + \Delta x) - \cos x] + \Delta x \cos (x + \Delta x)$$

$$= x(-2)\sin\frac{2x + \Delta x}{2}\sin\frac{\Delta x}{2} + \Delta x\cos(x + \Delta x)$$

$$\Rightarrow \qquad \frac{\Delta y}{\Delta x} = -2x\sin\frac{2x + \Delta x}{2}\sin\frac{\left(\frac{\Delta x}{2}\right)}{\Delta x} + \cos(x + \Delta x)$$

$$\therefore \qquad \frac{dy}{dx} = \lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} - x\sin\frac{2x + \Delta x}{2}\lim_{\Delta x \to 0}\sin\frac{\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} + \lim_{\Delta x \to 0}\cos(x + \Delta x)$$

$$= - x \sin x + \cos x$$

**S26.** Given, $\qquad y = x \sin x.$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $\qquad y + \Delta y = (x + \Delta x) \sin (x + \Delta x)$

$$\Rightarrow \qquad \Delta y = (x + \Delta x) \sin (x + \Delta x) - x \sin x$$

$$= x[\sin (x + \Delta x) - \sin x] + \Delta x \sin (x + \Delta x)$$

$$= x\cdot 2\cos\frac{2x + \Delta x}{2}\sin\frac{\Delta x}{2} + \Delta x\sin(x + \Delta x)$$

$$\Rightarrow \qquad \frac{\Delta y}{\Delta x} = 2x\cos\frac{2x + \Delta x}{2}\sin\frac{\left(\frac{\Delta x}{2}\right)}{\Delta x} + \sin(x + \Delta x)$$

$$\therefore \qquad \frac{dy}{dx} = \lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} x\cos\frac{2x + \Delta x}{2}\lim_{\Delta x \to 0}\sin\frac{\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}} + \lim_{\Delta x \to 0}\sin(x + \Delta x)$$

$$= x\cos x \cdot 1 + \sin x = x\cos x + \sin x$$

**S27.** Given, $y = \cot x.$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $y + \Delta y = \cot (x + \Delta x)$

$$\Rightarrow \quad \Delta y = \cot (x + \Delta x) - \cot x$$

$$= \frac{\cos (x + \Delta x)}{\sin (x + \Delta x)} - \frac{\cos x}{\sin x} = \frac{\sin x \cos (x + \Delta x) - \cos x \sin (x + \Delta x)}{\sin x \sin (x + \Delta x)}$$

$$= \frac{\sin [x - (x + \Delta x)]}{\sin x \sin (x + \Delta x)}$$

$$\Rightarrow \quad \frac{\Delta y}{\Delta x} = -\frac{\sin \Delta x}{\Delta x} \cdot \frac{1}{\sin x \sin (x + \Delta x)}$$

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} \lim_{\Delta x \to 0} \frac{1}{\sin x \sin (x + \Delta x)}$$

$$= -1 \cdot \frac{1}{\sin x \sin x} = -\operatorname{cosec}^2 x.$$

**S28.** Given, $y = \operatorname{cosec} x.$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $y + \Delta y = \operatorname{cosec} (x + \Delta x)$

$$\Rightarrow \quad \Delta y = \operatorname{cosec} (x + \Delta x) - \operatorname{cosec} x$$

$$= \frac{1}{\sin (x + \Delta x)} - \frac{1}{\sin x} = \frac{\sin x - \sin (x + \Delta x)}{\sin x \sin (x + \Delta x)}$$

$$= \frac{2\cos \left(\frac{2x + \Delta x}{2}\right) \sin \left(\frac{-\Delta x}{2}\right)}{\sin x \sin (x + \Delta x)}$$

$$\Rightarrow \quad \frac{\Delta y}{\Delta x} = -\frac{\cos \left(\frac{2x + \Delta x}{2}\right)}{\sin x \sin (x + \Delta x)} \cdot \frac{\sin \left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2\cos\left(\frac{2x+\Delta x}{2}\right)}{\sin x \sin(x+\Delta x)} \lim_{\Delta x \to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$= \frac{-\cos x}{\sin x \sin x} \cdot 1 = -\operatorname{cosec} x \cot x.$$

**S29.** Given, $y = \sec x.$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $y + \Delta y = \sec(x + \Delta x)$

$\Rightarrow$ $\Delta y = \sec(x + \Delta x) - \sec x$

$$= \frac{1}{\cos(x+\Delta x)} - \frac{1}{\cos x} = \frac{\cos x - \cos(x+\Delta x)}{\cos x \cos(x+\Delta x)}$$

$$= \frac{2\sin\left(\frac{2x+\Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)}{\cos x \cos(x+\Delta x)}$$

$\Rightarrow$ $$\frac{\Delta y}{\Delta x} = \frac{\sin\left(\frac{2x+\Delta x}{2}\right)}{\cos x \cos(x+\Delta x)} \cdot \frac{\sin\left(\frac{\Delta x}{2}\right)}{\left(\frac{\Delta x}{2}\right)}$$

$\therefore$ $$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin\left(\frac{2x+\Delta x}{2}\right)}{\cos x \cos(x+\Delta x)} \cdot \lim_{\Delta x \to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\left(\frac{\Delta x}{2}\right)}$$

$$= \frac{\sin x}{\cos x \cos x} \times 1 = \sec x \tan x.$$

**S30.** Given, $y = \tan x.$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $y + \Delta y = \tan(x + \Delta x)$

$\Rightarrow$ $\Delta y = \tan(x + \Delta x) - \tan x$

$$= \frac{\sin(x+\Delta x)}{\cos(x+\Delta x)} - \frac{\sin x}{\cos x}$$

$$= \frac{\sin(x+\Delta x)\cos x - \cos(x+\Delta x)\sin x}{\cos x \cos(x+\Delta x)}$$

$$= \frac{\sin(x+\Delta x - x)}{\cos x \cos(x+\Delta x)}$$

$\Rightarrow$ $$\frac{\Delta y}{\Delta x} = \frac{\sin \Delta x}{\Delta x} \cdot \frac{1}{\cos x \cos(x+\Delta x)}$$

$\therefore$ $$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} \lim_{\Delta x \to 0} \frac{1}{\cos x \cos(x+\Delta x)}$$

$$= 1 \cdot \frac{1}{\cos x \cdot \cos x} = \sec^2 x.$$

**S31.** Given, $y = \cos x$.

Let $\Delta x$ be a small change in $x$ and $\Delta y$ be the corresponding change in $y$.

Then, $y + \Delta y = \cos(x + \Delta x)$

$\Rightarrow$ $$\Delta y = \cos(x + \Delta x) - \cos x$$

$$= -2\sin\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}$$

$\Rightarrow$ $$\frac{\Delta y}{\Delta x} = -\sin\frac{2x+\Delta x}{2} \cdot \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$\therefore$ $$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= -\lim_{\Delta x \to 0} \sin\frac{2x+\Delta x}{2} \lim_{\Delta x \to 0} \frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$= -(\sin x) \cdot 1 = -\sin x$$

**S32.** Given, $y = \sin x$.

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $y + \Delta y = \sin(x + \Delta x)$

$\Rightarrow$ $$\Delta y = \sin(x + \Delta x) - \sin x$$

$$= 2\cos\frac{2x+\Delta x}{2}\sin\frac{\Delta x}{2}$$

$$\Rightarrow \qquad \frac{\Delta y}{\Delta x} = \cos\frac{2x+\Delta x}{2}\cdot\frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$\therefore \qquad \frac{dy}{dx} = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\cos\frac{2x+\Delta x}{2}\cdot\lim_{\Delta x\to 0}\frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}$$

$$= (\cos x)\cdot 1 = \cos x.$$

**S33.** Let $\quad f(x) = (x^2+1)(x-5),$

$$f(x) = x^3 + x - 5x^2 - 5$$

$$f(x+h) = (x+h)^3 - 5(x+h)^2 + (x+h) - 5$$

$$f(x+h) - f(x) = x^3 + h^3 + 3x^2h + 3xh^2 - 5x^2 - 10xh + x + h - 5 - x^3 - x + 5x^2 + 5$$

divided by $h$

$$\frac{f(x+h)-f(x)}{h} = \frac{h^3+3x^2h+3xh^2-5h^2-10xh+h}{h}$$

$$\frac{f(x+h)-f(x)}{h} = h^2 + 3x^2 + 3xh - 5h - 10x + 1$$

Taking limit on both sides as $h\to 0$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(h^2+3x^2+3xh-5h-10x+1)$$

$$f'(x) = 3x^2 + 10x + 1.$$

**S34.** Let $\quad f(x) = (x+1)(x+2)(x+3)$

$$= (x^2 + x + 2x + 2)(x+3)$$

$$= (x^2 + 3x + 2)(x+3)$$

$$= x^3 + 6x^2 + 11x + 6$$

$$f(x+h) = (x+h)^3 + 6(x+h)^2 + 11(x+h) + 6$$

$$f(x+h) - f(x) = h^3 + 3x^2h + 3xh^2 + 6h^2 + 12xh + 11h$$

Taking limit on both sides as $h\to 0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} [h^2 + 3x^2 + 3xh + 6h + 12x + 11]$$

$$f'(x) = 3x^2 + 12x + 11.$$

**S35.** Let $f(x) = (x + 1)(x + 2) \Rightarrow f(x) = x^2 + 3x + 2$ ... (i)

$$f(x + h) = (x + h)^2 + 3(x + h) + 2 \quad \text{... (ii)}$$

Subtracting (i) and (ii), we get

$$f(x + h) - f(x) = h^2 + 2xh + 3h$$

divided by $h$

$$\frac{f(x+h) - f(x)}{h} = \frac{h^2 + 2xh + 3h}{h} = h + 2x + 3$$

Taking limit on both sides as $h \to 0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (h + 2x + 3) \quad \Rightarrow \quad f'(x) = 2x + 3.$$

**S36.** Let $f(x) = x^4 + 1$

$$f(x + h) - f(x) = (x + h)^4 + 1 - x^4 - 1$$
$$= h^4 + 2x^2h^2 + 4x^2h^2 + 4x^3h + 4xh^3$$
$$= h^4 + 6x^2h^2 + 4x^3h + 4xh^3$$

divided by $h$

$$\frac{f(x+h) - f(x)}{h} = \frac{h^4 + 6x^2h^2 + 4x^3h + 4xh^3}{h}$$

$$= h^3 + 6x^2h + 4x^3 + 4xh^2$$

Taking limit on both sides as $h \to 0$

$$\Rightarrow \quad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (h^3 + 6x^2h + 4x^3 + 4xh^2).$$

$$f'(x) = 4x^3.$$

**S37.** Let $f(x) = x^3$ ... (i)

$$f(x + h) = (x + h)^3 = x^3 + (h)^3 + 3x^2(h) + 3x(h)^2 \quad \text{... (ii)}$$

Subtracting Eq. (i) from Eq. (ii), we get

$$f(x + h) - f(x) = (h)^3 + 3x^2(h) + 3x(h)^2$$

Dividing by $h$, we get

$\Rightarrow \qquad \dfrac{f(x+h)-f(x)}{h} = \dfrac{(h)^3+3x^2(h)+3x(h)^2}{h}$

Taking limit on both sides, as $h \to 0$.

$\Rightarrow \qquad \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{(h)[(h)^2+3x^2+3x(h)]}{(h)}$

$\Rightarrow \qquad f'(x) = 0 + 3x^2 + 0 = 3x^2.$ **Ans.**

**S38.** We have, $\quad f(x) = \dfrac{1}{x^2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left(\dfrac{1}{x+h}\right)^2 - \dfrac{1}{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{1}{x^2+2xh+h^2} - \dfrac{1}{x^2}}{h} = \lim_{h\to 0} \frac{\dfrac{x^2-(x^2+2hx+h^2)}{x^2(x^2+2hx+h^2)}}{h}$$

$$= \lim_{h\to 0} \frac{-2hx-h^2}{hx^2(x^2+2hx+h^2)} = \lim_{h\to 0} \frac{-2x-h}{x^2(x^2+2hx+h^2)}$$

$$= \lim_{h\to 0} \frac{-2x-h}{x^4+4hx^3+x^2h^2} = \frac{-2x}{x^4} = -\frac{2}{x^3}.$$ **Ans.**

**S39.** We have, $\quad f(x) = (x-1)(x-2)$

$\Rightarrow \qquad f(x) = x^2 - 3x + 2$

$\Rightarrow \qquad f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{[(x+h)^2-3(x+h)+2]-[x^2-3x+2]}{h}$

$$= \lim_{h\to 0} \frac{[x^2+2hx+h^2-3x-3h+2]-[x^2-3x+2]}{h} = \lim_{h\to 0} \frac{2hx+h^2-3h}{h}$$

$$= \lim_{h\to 0} \frac{h[2x+h-3]}{h} = \lim_{h\to 0} [2x+h-3] = 2x-3.$$ **Ans.**

**S40.** Given, $\qquad y = \dfrac{\cos x}{x}.$

Let $\Delta x$ be a small change in $x$ and $\Delta y$ the corresponding change in $y$.

Then, $\qquad y + \Delta y = \dfrac{\cos(x+\Delta x)}{(x+\Delta x)}$

$\Rightarrow \qquad \Delta y = \dfrac{\cos(x+\Delta x)}{x+\Delta x} - \dfrac{\cos x}{x} = \dfrac{x\cos(x+\Delta x)-(x+\Delta x)\cos x}{x(x+\Delta x)}$

$$\Rightarrow \qquad \frac{\Delta y}{\Delta x} = \frac{x[\cos(x+\Delta x) - \cos x] - \Delta x \cos x}{x(x+\Delta x)\Delta x}$$

$$= \frac{1}{x+\Delta x}\frac{\cos(x+\Delta x) - \cos x}{\Delta x} - \frac{\cos x}{x(x+\Delta x)}$$

$$\therefore \qquad \frac{dy}{dx} = \lim_{\Delta x \to 0}\frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0}\frac{1}{x+\Delta x}\lim_{\Delta x \to 0}\frac{\cos(x+\Delta x) - \cos x}{\Delta x} - \lim_{\Delta x \to 0}\frac{\cos x}{x(x+\Delta x)}$$

$$= \frac{1}{x}\cdot\frac{d}{dx}(\cos x) - \frac{\cos x}{x^2} = \frac{1}{x}\cdot(-\sin x) - \frac{\cos x}{x^2}$$

$$= -\frac{\sin x}{x} - \frac{\cos x}{x^2}.$$

**S41.** We have, $f(x) = x^2 - 2$

$$f'(10) = \lim_{h \to 0}\frac{f(10+h) - f(10)}{h}$$

$$= \lim_{h \to 0}\frac{[(10+h)^2 - 2] - [10^2 - 2]}{h}$$

$$= \lim_{h \to 0}\frac{(100 + 20h + h^2 - 2) - (100 - 2)}{h}$$

$$= \lim_{h \to 0}\frac{(h^2 + 20h + 98) - (98)}{h}$$

$$= \lim_{h \to 0}(h + 20) = 20.$$

**S42.** We first find the derivatives of $f(x)$ at $x = -1$ and $x = 0$. We have

$$f'(-1) = \lim_{h \to 0}\frac{f(-1+h) - f(-1)}{h}$$

$$= \lim_{h \to 0}\frac{[2(-1+h)^2 + 3(-1+h) - 5] - [2(-1)^2 + 3(-1) - 5]}{h}$$

$$= \lim_{h \to 0}\frac{2h^2 - h}{h} = \lim_{h \to 0}(2h - 1) = 2(0) - 1 = -1$$

and

$$f'(0) = \lim_{h \to 0}\frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0}\frac{[2(0+h)^2 + 3(0+h) - 5] - [2(0)^2 + 3(0) - 5]}{h}$$

$$= \lim_{h \to 0}\frac{2h^2 + 3h}{h} = \lim_{h \to 0}(2h + 3) = 2(0) + 3 = 3$$

Clearly, $f'(0) + 3f'(-1) = 0$.

**S43.** Let $f(x) = \sin(x+1)$

$\therefore$ $$f'(x) = \lim_{h \to 0} \frac{\sin(x+h+1) - \sin(x+1)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(x+1+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$= \lim_{h \to 0} \cos\left[(x+1)+\frac{h}{2}\right] \cdot \frac{\sin\frac{h}{2}}{\frac{h}{2}} = \cos(x+1).$$ **Ans.**

**S44.** Let, $f(x) = (-x)^{-1} = -\frac{1}{x}$

$\therefore$ $$f'(x) = \lim_{h \to 0} \frac{-\frac{1}{x+h} - \left(-\frac{1}{x}\right)}{h} = \lim_{h \to 0} \frac{-\frac{1}{x+h} + \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{[x+h]-x}{h(x+h)x} = \lim_{h \to 0} \frac{h}{hx(x+h)} = \lim_{h \to 0} \frac{1}{x(x+h)} = \frac{1}{x^2}.$$ **Ans.**

**S45.** Let $y = \tan x^2$

Let $h$ be a small increment in $x$.

$\therefore$ $f(x+h) = \tan(x+h)^2$

$$f(x+h) - f(x) = \tan(x+h)^2 - \tan x^2 = \frac{\sin(x+h)^2}{\cos(x+h)^2} - \frac{\sin x^2}{\cos x^2}$$

$$= \frac{\sin(x+h)^2\cos x^2 - \cos(x+h)^2\sin x^2}{\cos(x+h)^2\cos x^2}$$

$$= \frac{\sin(x+h)^2 - x^2}{\cos(x+h)^2\cos x^2}$$ $[\because \sin A\cos B - \cos A\sin B = \sin(A-B)]$

Now, $$\frac{f(x+h)-f(x)}{h} = \frac{\sin(2x+h)h}{\cos(x+h)^2\cos x^2} \cdot \frac{1}{h} = \frac{\sin(2x+h)h}{(2x+h)h} \times \frac{2x+h}{\cos(x+h)^2\cos x^2}$$

Now, proceed to limits as $h \to 0$, we get

$\therefore$ $$f'(x) = 1 \cdot \frac{2x}{\cos x^2\cos x^2} = 2x\sec^2 x^2.$$ **Ans.**

**S46.** Let $f(x) = (ax+b)^n$

$$f(x+h) = (ax+b+ah)^n$$

$$f(x+h) - f(x) = (ax+b+ah)^n - (ax+b)^n$$

divided by $h$

$$\frac{f(x+h)-f(x)}{h} = \frac{(ax+b+ah)^n-(ax+b)^n}{h}$$

$$\frac{f(x+h)-f(x)}{h} = a\,\frac{[(ax+b+ah)^n-(ax+b)^n]}{ah}$$

Taking limit on both sides as $h \to 0$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = a\lim_{h\to 0}\frac{(ax+b+ah)^n-(ax+b)^n}{ah}$$

$$= \lim_{(ax+b+ah)\to(ax+b)} a\,\frac{(ax+b+ah)^n-(ax+b)^n}{(ax+b+ah)-(ax+b)}$$

$$= a\,.\,n\,(ax+b)^{n-1} \qquad \left[\because\ \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

**S47.** Let

$$f(x) = \frac{x+1}{x-1}$$

$$\frac{df}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{(x+h)+1}{(x+h)-1}-\dfrac{x+1}{x-1}}{h}$$

$$= \lim_{h\to 0}\frac{(x-1)[(x+h)+1]-[(x+h)-1](x+1)}{h[(x+h)-1][x-1]}$$

$$= \lim_{h\to 0}\frac{x^2+xh+x-x-h-1-(x^2+xh-x+x+h-1)}{h[x^2+xh-x-x-h+1]}$$

$$= \lim_{h\to 0}\frac{x^2+xh-h-1-x^2-xh-h+1}{h[x^2+xh-2x-h+1]}$$

$$= \lim_{h\to 0}\frac{-2h}{h[x^2+xh-2x-h+1]}$$

$$= \lim_{h\to 0}\frac{-2}{x^2+xh-2x-h+1}$$

$$= \lim_{h\to 0}\frac{-2}{x^2-2x+1} = \frac{-2}{(x-1)^2}.$$

**S48.** We have,

$$f(x) = 99x.$$

$$f'(100) = \lim_{h \to 0} \frac{f(100+h) - f(100)}{h}$$

$$= \lim_{h \to 0} \frac{99(100+h) - 99(100)}{h}$$

$$= \lim_{h \to 0} \frac{9900 + 99h - 9900}{h}$$

$$= \lim_{h \to 0} 99 = 99.$$

**S49.** Let

$$f(x) = x^2 - 19x + 18 \quad \text{... (i)}$$

$$\Rightarrow \quad f(x+h) = (x+h)^2 - 19(x+h) + 18$$

$$= x^2 + (h)^2 + 2x(h) - 19x - 19h + 18 \quad \text{... (ii)}$$

Subtracting (i) from (ii), we get

$$f(x+h) - f(x) = x^2 + (h)^2 + 2x(h) - 19x - 19h + 18 - x^2 + 19x - 18$$

$$= (h)^2 + 2x(h) - 19h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(h)^2 + 2x(h) - 19(h)}{h}$$

Taking limit as $h \to 0$ on both sides, we get

$$\lim_{\delta x \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{h[(h) + 2x - 19]}{h} = \lim_{h \to 0} [h + 2x - 19]$$

$$\Rightarrow \quad f'(x) = 0 + 2x - 19$$

$$\Rightarrow \quad f'(x) = 2x - 19$$

Therefore,

$$f'(2) = 2(2) - 19 = 4 - 19 = -15$$

$$f'(1) = 2(1) - 19 = 2 - 19 = -17$$

$$f'(10) = 2(10) - 19 = 20 - 19 = 1$$

**Ans.**

**S50.** Let

$$f(x) = x + \frac{1}{x} = \frac{x^2 + 1}{x} \quad \text{... (i)}$$

$$f(x+h) = \frac{(x+h)^2 + 1}{x+h} \quad \text{... (ii)}$$

Subtracting Eq. (i) from Eq. (ii), we get

$$f(x+h) - f(x) = \frac{(x+h)^2 + 1}{x+h} - \frac{x^2 + 1}{x}$$

$$= \frac{x(x^2 + (h)^2 + 2xh + 1) - (x^2 + 1)(x+h)}{(x+h)x}$$

$$= \frac{x^2 + x(h)^2 + 2x^2h + x - x^3 - x^2h - x - h}{(x+h)x}$$

Dividing by $h$, we get

$$\Rightarrow \quad \frac{f(x+h) - f(x)}{h} = \frac{h[x(h) + x^2 - 1]}{h[x+h]x}$$

Taking limit on both sides, we get

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[x(h) + x^2 - 1]}{x(x+h)}$$

$$\Rightarrow \quad f'(x) = \frac{0 + x^2 - 1}{x(x+0)} = \frac{x^2 - 1}{x^2} = 1 - \frac{1}{x^2}.$$ **Ans.**

**S51.** We have, $f(x) = 3x^3 + 2x^2 + 3x - 6$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{[3(x+h)^3 + 2(x+h)^2 + 3(x+h) - 6] - [3x^3 + 2x^2 + 3x - 6]}{h}$$

$$= \lim_{h \to 0} \frac{[3(x^3 + 3x^2h + 3h^2x + h) + 2(x^2 + 2hx + h^2) + 3(x+h) - 6] - [3x^3 + 2x^2 + 3x - 6]}{h}$$

$$= \lim_{h \to 0} \frac{9x^2h + 9h^2x + 3h^3 + 4hx + 2h^2 + 3h}{h} = \lim_{h \to 0} \frac{h[9x^2 + 9hx + 3h^2 + 4x + 2h + 3]}{h}$$

$$= \lim_{h \to 0} [9x^2 + 9hx + 3h^2 + 4x + 2h + 3].$$

$\Rightarrow \quad f'(x) = 9x^2 + 4x + 3$

$\Rightarrow \quad f'(-1) = 9 - 4 + 3 = 8$

$f'(0) = 3$

Now, $8f'(0) - 3f'(-1) = 8(3) - 3(8) = 24 - 24$

$\Rightarrow \quad 8f'(0) - 3f'(-1) = 0.$ **Ans.**

**S52.** Let $f(x) = \frac{ax^2 + bx + c}{\sqrt{x}}$

$$\Rightarrow \quad f(x) = \frac{ax^2}{\sqrt{x}} + \frac{bx}{\sqrt{x}} + \frac{c}{\sqrt{x}} = ax^{3/2} + bx^{1/2} + cx^{-1/2}$$

$$f(x+h) = a(x+h)^{3/2} + b(x+h)^{1/2} + c(x+h)^{-1/2}$$

$$f(x+h) - f(x) = a(x+h)^{3/2} + b(x+h)^{1/2} + c(x+h)^{-1/2} - (ax^{3/2} + bx^{1/2} + cx^{-1/2})$$

$$= a[(x+h)^{3/2} - x^{3/2}] + b[(x+h)^{1/2} - x^{1/2}] + c[(x+h)^{-1/2} - x^{-1/2}]$$

$$\frac{f(x+h)-f(x)}{h} = \frac{a[(x+h)^{3/2} - x^{3/2}]}{h} + \frac{b[(x+h)^{1/2} - x^{1/2}]}{h} + \frac{c[(x+h)^{-1/2} - x^{-1/2}]}{h}$$

Taking limit on both sides as $h \to 0$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{a[(x+h)^{3/2} - x^{3/2}]}{h} + \lim_{h\to 0}\frac{b[(x+h)^{1/2} - x^{1/2}]}{h} + \lim_{h\to 0}\frac{c[(x+h)^{-1/2} - x^{-1/2}]}{h}$$

$$= a\lim_{h\to 0}\frac{[(x+h)^{3/2} - x^{3/2}]}{(x+h)-x} + b\lim_{h\to 0}\frac{[(x+h)^{1/2} - x^{1/2}]}{(x+h)-x} + c\lim_{h\to 0}\frac{[(x+h)^{-1/2} - x^{-1/2}]}{(x+h)-x}$$

$$= \frac{3}{2}ax^{1/2} + \frac{1}{2}bx^{-1/2} - \frac{1}{2}cx^{-3/2} = \frac{3a\sqrt{x}}{2} + \frac{b}{2\sqrt{x}} - \frac{c}{2x^{3/2}}.$$

**S53.** Let

$$f(x) = \sqrt{x+1}$$

$$f(x+h) = \sqrt{x+h+1}$$

$$f(x+h) - f(x) = \sqrt{x+h+1} - \sqrt{x+1}$$

divided by $h$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

Rationalize the numerator

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \times \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$\Rightarrow \quad \frac{f(x+h)-f(x)}{h} = \frac{x+h+1-x-1}{h[\sqrt{x+h+1} + \sqrt{x+1}]} = \frac{h}{\sqrt{x+h+1} + \sqrt{x+1}}$$

Taking limit on both sides as $h \to 0$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$\Rightarrow \quad f'(x) = \lim_{h\to 0}\frac{1}{\sqrt{x+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}.$$

**S54.** Let

$$f(x) = \frac{2-x}{4+3x}$$

$$f(x+h) = \frac{2-(x+h)}{4+3(x+h)}$$

$$f(x+h)-f(x) = \frac{2-(x+h)}{4+3(x+h)} - \frac{2-x}{4+3x}$$

$$= \frac{[2-(x+h)][4+3x]-(2-x)[4+3(x+h)]}{[4+3(x+h)][4+3x]}$$

$$= \frac{8-4x+6x-3x^2-4h-3xh-8+4x-6x+3x^2-6h+3xh}{(4+3x+3h)(4+3x)}$$

$$f(x+h)-f(x) = \frac{-10h}{(4+3x+3h)(4+3x)}$$

divided by $h$

$$\frac{f(x+h)-f(x)}{h} = \frac{-10}{(4+3x+3h)(4+3x)}$$

Taking limit on both sides as $h \to 0$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{-10}{(4+3x+3h)(4+3x)} \Rightarrow f'(x) = \frac{-10}{(4+3x)^2}.$$

**S55.** Let $f(x) = x^3 - 3x^2 + 3x - 1$ ... (i)

$\Rightarrow$ $f(x+h) = (x+h)^3 - 3(x+h)^2 + 3(x+h) - 1$

$= x^3 + (h)^3 + 3x^2(h) + 3(h)^2x - 3[x^2 + (h)^2 + 2xh] + 3x + 3h - 1$

$= x^3 + (h)^3 + 3x^2(h) + 3(h)^2x - 3x^2 - 3(h)^2 - 6xh + 3x + 3h - 1$ ... (ii)

Subtracting (i) from (ii), we get

$f(x+h) - f(x) = x^3 + (h)^3 + 3x^2(h) + 3(h)^2x - 3x^2 - 3(h)^2 - 6xh + 3x + 3h - 1$

$- x^3 + 3x^2 - 3x + 1$

$= (h)^3 + 3x^2(h) + 3h^2x - 3(h)^2 - 6xh + 3h$

Dividing by $h$, we get

$\Rightarrow$ $$\frac{f(x+h)-f(x)}{h} = \frac{(h)[(h)^2+3x^2+3(h)x-3(h)-6x+3]}{h}$$

Taking limit on both sides as $h \to 0$

$\Rightarrow$ $$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}[(h)^2+3x^2+3(h)x-3(h)-6x+3]$$

$\Rightarrow$ $f'(x) = 0 + 3x^2 + 3(0)x - 3(0) - 6x + 3$

$\Rightarrow$ $f'(x) = 3x^2 - 6x + 3$

Given that $f'(a) = 0$

$\Rightarrow$ $3a^2 - 6a + 3 = 0$

$\Rightarrow$ $3[a^2 - 2a + 1] = 0$

$$\Rightarrow \qquad 3(a-1)^2 = 0$$

$$\Rightarrow \qquad a = 1$$ **Ans.**

**S56.** We have, $$f(x) = \cos\left(x - \frac{\pi}{8}\right)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\cos\left(x + h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{h}$$

$$= \lim_{h \to 0} \frac{2\sin\frac{1}{2}\left(x + h - \frac{\pi}{8} + x - \frac{\pi}{8}\right)\sin\frac{1}{2}\left(x - \frac{\pi}{8} - x - h\frac{\pi}{8}\right)}{h}$$

$$= \lim_{h \to 0} \frac{2\sin\left(x + \frac{h}{2} - \frac{\pi}{8}\right)\sin\left(\frac{-h}{2}\right)}{h} = -\lim_{h \to 0} \frac{\sin\left(x + \frac{h}{2} - \frac{\pi}{8}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{h \to 0} \sin\left(x + \frac{h}{2} - \frac{\pi}{8}\right) \cdot \lim_{\frac{h}{2} \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} = -\lim_{h \to 0} \sin\left(x + \frac{h}{2} - \frac{\pi}{8}\right) \cdot (1)$$

$$= -\sin\left(x - \frac{\pi}{8}\right)$$ **Ans.**

**S57.** Let $f(x) = \sec\left(\frac{x}{2} - 1\right)$, $f(x+h) = \sec\left(\frac{x+h}{2} - 1\right)$, $f(x+h) - f(x) = \sec\left(\frac{x+h}{2} - 1\right) - \sec\left(\frac{x}{2} - 1\right)$

$$f(x+h) - f(x) = \frac{1}{\cos\left(\frac{x+h}{2} - 1\right)} - \frac{1}{\cos\left(\frac{x}{2} - 1\right)}$$

$$f(x+h) - f(x) = \frac{\cos\left(\frac{x}{2} - 1\right) - \cos\left(\frac{x+h}{2} - 1\right)}{\cos\left(\frac{x+h}{2} - 1\right)\cos\left(\frac{x}{2} - 1\right)}$$

$$f(x+h) - f(x) = \frac{2\sin\frac{\left(\frac{x + 2 + x + h - 2}{2}\right)}{2}\sin\frac{\left(\frac{x - 2 - x - h + 2}{2}\right)}{2}}{\cos\left(\frac{x+h}{2} - 1\right)\cos\left(\frac{x}{2} - 1\right)}$$

Dividing by $h$, we get

$$\frac{f(x+h) - f(x)}{h} = \frac{-2\sin\left(\frac{2x + h - 4}{4}\right)\sin\left(-\frac{h}{2}\right)}{h\cos\left(\frac{2x+h}{2} - 1\right)\sin\left(\frac{x}{2} - 1\right)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2\sin\left(\frac{2x+h-4}{4}\right)\sin\left(-\frac{h}{2}\right)}{h\cos\left(\frac{2x+h}{2}-1\right)\cos\left(\frac{x}{2}-1\right)} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2\sin\left(\frac{2x+h-4}{4}\right)}{\cos\left(\frac{x}{2}-1\right)\cos\left(\frac{x+h}{2}-1\right)} - \lim_{\frac{h}{4}\to 0}\frac{\frac{1}{4}\sin\left(\frac{h}{4}\right)}{\frac{h}{4}}$$

$$f'(x) = \frac{2\sin\left(\frac{2x-4}{4}\right)}{\cos^2\left(\frac{x}{2}-1\right)}\times\frac{1}{4}\times 1 = \frac{1}{2}\sin\left(\frac{x}{2}-1\right)\times\frac{1}{\cos^2\left(\frac{x}{2}-1\right)\cos\left(\frac{x}{2}-1\right)}$$

$$= \frac{1}{2}\tan\left(\frac{x}{2}-1\right)\sec\left(\frac{x}{2}-1\right).$$

**S58.** Let $f(x) = \cos(x^2+1), \quad f(x+h) = \cos[(x+h)^2+1]$

$$f(x+h)-f(x) = \cos[(x+h)^2+1] - \cos(x^2+1)$$

$$= -2\sin\left(\frac{(x+h)^2+1+x^2+1}{2}\right)\cdot\sin\left(\frac{(x+h)^2+1-(x^2+1)}{2}\right)$$

$$= -2\sin\left(\frac{x^2+h^2+2xh+1+x^2+1}{2}\right)\cdot\sin\left(\frac{x^2+h^2+2xh+1-x^2-1}{2}\right)$$

$$= -2\sin\left(\frac{2x^2+h^2+2xh+2}{2}\right)\sin\left(\frac{h(h+2x)}{2}\right)$$

$$f(x+h)-f(x) = -\frac{2\sin\left(\frac{2x^2+2+2xh+h^2}{2}\right)\sin\left(\frac{h(h+2x)}{2}\right)}{h}$$

$$= -\frac{2\sin\left(\frac{2x^2+2xh+2+h^2}{2}\right)\sin\left(\frac{h(h+2x)}{2}\right)}{h}$$

Taking limit on both sides as $h \to 0$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{-2\sin\left(\frac{2x^2+2xh+2+h^2}{2}\right)\sin\frac{h(h+2x)}{2}}{h}$$

$$f'(x) = \lim_{h\to 0}\sin\left(\frac{2x^2+2xh+2+h^2}{2}\right)\lim_{\frac{h(h+2x)}{2}\to 0}\frac{\sin\left(\frac{h(h+2x)}{2}\right)}{\frac{h(h+2x)}{2}}\cdot\frac{(h+2x)}{2}$$

$$= -2 \cdot \sin(x^2+1) \cdot 1 \cdot x = -2x \sin(x^2+1).$$

**S59.** Let
$$f(x) = \frac{\sin x}{x} \quad \text{... (i)}$$
$$\Rightarrow \quad f(x+h) = \frac{\sin(x+h)}{(x+h)} \quad \text{... (ii)}$$

Subtracting (i) from (ii) and dividing by $h$, we get

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{\sin(x+h)}{(x+h)} - \frac{\sin x}{x}}{h}$$

Taking limit on both sides as $h \to 0$, we have

$$\Rightarrow \quad \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{x\sin(x+h)-(x+h)\sin x}{h(x+h)x}$$

$$\Rightarrow \quad f'(x) = \lim_{h\to 0}\frac{x[\sin(x+h)-\sin x]-h\sin x}{h\cdot x(x+h)}$$

$$= \lim_{h\to 0}\frac{x\left[2\sin\left(\frac{x+h-x}{2}\right)\cos\left(\frac{x+h+x}{2}\right)\right]-h\sin x}{h\cdot x\cdot(x+h)}$$

$$\Rightarrow \quad f'(x) = \lim_{\frac{h}{2}\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\cdot\lim_{h\to 0}\frac{\cos\left(\frac{h}{2}\right)}{(x+h)} - \lim_{h\to 0}\frac{\sin x}{x(x+h)}$$

$$= (1)\cdot\frac{\cos x}{x} - \lim_{h\to 0}\frac{\sin x}{x(x+h)} = \frac{\cos x}{x} - \frac{\sin x}{x\cdot x} = \frac{\cos x}{x} - \frac{\sin x}{x^2}. \quad \textbf{Ans.}$$

**S60.** Let $f(x) = \frac{\sin x}{x-6}$, $\quad f(x+h) = \frac{\sin(x+h)}{x+h-6}$, $\quad f(x+h)-f(x) = \frac{\sin(x+h)}{x+h-6} - \frac{\sin x}{x-6}$

$$f(x+h)-f(x) = \frac{(x-6)\sin(x+h)-(x+h-6)\sin x}{(x+h-6)(x-6)}$$

$$= \frac{(x-6)[\sin(x+h)-\sin x]-h\sin x}{(x+h-6)(x-6)}$$

$$= \frac{(x-6)\times 2\cos\left(\frac{x+h+x}{2}\right)\cdot \sin\left(\frac{x+h-x}{2}\right)}{(x+h-6)(x-6)} - \frac{h\sin x}{(x+h-6)(x-6)}$$

$$\left[\because \quad \sin C - \sin D = 2\cos\frac{C+D}{2}\cdot\sin\frac{C-D}{2}\right]$$

$$= \frac{(x-6)\,2\cos\left(\frac{2x+h}{2}\right)\sin\frac{h}{2}}{(x+h-6)(x-6)} - \frac{h\sin x}{(x+h-6)(x-6)}$$

Dividing by $h$, we get

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x-6)\cos\left(\frac{2x+h}{2}\right)\sin\frac{h}{2}}{h(x+h-6)(x-6)} - \frac{\sin x}{(x+h-6)(x-6)}$$

Taking limit on both sides as $h \to 0$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2(x-6)\cos\left(\frac{2x+h}{2}\right)\sin\frac{h}{2}}{h(x+h-6)(x-6)} - \lim_{h\to 0}\frac{\sin x}{(x+h-6)(x-6)}$$

$$f'(x) = \lim_{h\to 0}\frac{2(x-6)\cos\frac{(2x+h)}{2}}{(x+h-6)(x-6)}\cdot \lim_{\frac{h}{2}\to 0}\frac{\frac{1}{2}\sin\frac{h}{2}}{\frac{h}{2}} - \frac{\sin x}{(x-6)^2}$$

$$= \frac{2(x-6)\cos x}{(x-6)^2}\times\frac{1}{2} - \frac{\sin x}{(x-6)^2} = \frac{(x-6)\cos x - \sin x}{(x-6)^2}.$$

Date: 12/10/2021

**Q1.** If $y = (x + a)^n$. Find $\frac{dy}{dx}$, where $n \in R$.

**Q2.** If $y = \frac{x+a}{x+b}$. Find $\frac{dy}{dx}$, where $a, b \in R$.

**Q3.** If $y = \sin(x + a)$. Find $\frac{dy}{dx}$.

**Q4.** If $y = (ax + b)^n$. Find $\frac{dy}{dx}$.

**Q5.** If $y = (x^2 + x + 1) \cdot \sin 2x$. Find $\frac{dy}{dx}$.

**Q6.** If $y = (x + 1)(x + 2)$. Find $\frac{dy}{dx}$.

**Q7.** If $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$. Find $\frac{dy}{dx}$.

**Q8.** Find $\frac{d}{dx}\left(x + \frac{1}{x}\right)^2, \quad x \neq 0$.

**Q9.** If $y = (2 + x)^2$, find $\frac{dy}{dx}$.

**Q10.** If $y = 5x^2$, find $\frac{dy}{dx}$.

**Q11.** If $y = 5 \cos x$, find $\frac{dy}{dx}$.

**Q12.** Compute the derivative of $f(x) = \sin 2x$.

**Q13.** Compute the derivative of $f(x) = \sin^2 x$.

**Q14.** Find the derivative of $f(x) = \frac{x+1}{x}$.

**Q15.** If $y = \sin^n x$. Find $\frac{dy}{dx}$, where $(n \in R)$.

**Q16.** If $y = x^2 \tan x$. Find $\frac{dy}{dx}$.

**Q17.** If $y = x^2 \sin x$. Find $\frac{dy}{dx}$.

**Q18.** If $y = (x + a) \cdot \tan 2x$. Find $\frac{dy}{dx}$, where $a \in R$.

**Q19.** If $y = \tan x^2$. Find $\frac{dy}{dx}$.

**Q20.** If $y = x^2 \sin 3x$. Find $\frac{dy}{dx}$.

**Q21.** Find the derivative of $x^5 \cot x$.

**Q22.** Compute the derivative of $g(x) = \cot x$.

**Q23.** Find the derivative of: $5 \sin x - 6 \cos x + 7$.

**Q24.** Find the derivative of: $5 \sec x + 4 \cos x$.

**Q25.** Find the derivative of: $\operatorname{cosec} x \cdot \cot x$.

**Q26.** Find the derivative of: $\sin(x + a)$.

**Q27.** Find the derivative of: $4\sqrt{x} - 2$.

**Q28.** Find the derivative of: $(x + a)$.

**Q29.** Find the derivative of $\dfrac{x^5 - \cos x}{\sin x}$.

**Q30.** Find the derivative of: $\operatorname{cosec} x$.

**Q31.** Find the derivative of: $x^5 (3 - 6x^{-9})$.

**Q32.** Find the derivative of: $\sec x$.

**Q33.** Find the derivative of: $\sin x \cos x$.

**Q34.** Find the derivative of: $2x - \dfrac{3}{4}$.

**Q35.** For some constants $a$ and $b$, find the derivative of: $(x - a)(x - b)$.

**Q36.** Find the derivative of $\dfrac{x + \cos x}{\tan x}$.

**Q37.** If $y = (ax)^m + b^m$. Find $\dfrac{dy}{dx}$.

**Q38.** If $y = 5 \cdot (\sin(1 + nx))$, find $\dfrac{dy}{dx}$ where $n \in R$.

**Q39.** If $y = x^n \tan x$. Find $\dfrac{dy}{dx}$.

**Q40.** If $y = \dfrac{x}{\cos x}$. Find $\dfrac{dy}{dx}$.

**Q41.** If $y = \dfrac{x}{1 + \cos x}$. Find $\dfrac{dy}{dx}$.

**Q42.** If $y = \dfrac{1}{1 + \sin x}$. Find $\dfrac{dy}{dx}$.

**Q43.** If $y = (x^2 - 5x + 6)(x^3 + 2)$. Find $\dfrac{dy}{dx}$.

**Q44.** If $y = (1 - \cos x)(1 + \tan x)$. Find $\dfrac{dy}{dx}$.

**Q45.** If $y = (x^2 - 5x + 6) \sec x$. Find $\dfrac{dy}{dx}$.

**Q46.** If $y = (1 + 2 \tan x)(5 + 4 \cos x)$. Find $\dfrac{dy}{dx}$.

**Q47.** If $y = \dfrac{ax + b}{cx + d}, (a, b, c, d \in R)$. Find $\dfrac{dy}{dx}$.

**Q48.** If $y = x^4 (\sin x - \cos x)$. Find $\dfrac{dy}{dx}$.

Q49. Find $\frac{dy}{dx}$, if y = cot x.

Q50. Differenciate $\frac{x^n - a^n}{x - a}$ w.r.t. *x*.

Q51. Find $\frac{dy}{dx}$.

If $y = \frac{1+\frac{1}{x}}{1-\frac{1}{x}}$

Q52. Differentiate the following function with respect to *x*:

$$\frac{(x-1)(x-2)}{(x-3)(x-4)}$$

Q53. Find the derivative of $x^n + ax^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$ for some fixed real number *a*.

Q54. If $f(x) = \frac{x^2+3x-9}{x^2-9x+3}$, find $f'(x)$.

Q55. If $y = \frac{x^2}{\sin x}$. Find $\frac{dy}{dx}$.

Q56. If $y = \frac{x}{1+\cot x}$. Find $\frac{dy}{dx}$.

Q57. If $y = \frac{x}{1+\tan x}$. Find $\frac{dy}{dx}$.

Q58. Find the derivative of: 2 tan *x* – 7 sec *x*.

Q59. Find the derivative of: 3 cot *x* + 5 cosec *x*.

Q60. Find the derivative of: $\frac{2}{x+1} - \frac{x^2}{3x-1}$.

Q61. Find the derivative of: $x^{-4}(3 - 4x^{-5})$.

Q62. Find the derivative of: $x^{-3}(5 + 3x)$.

Q63. Find the derivative of: $(5x^3 + 3x - 1)(x - 1)$.

Q64. For some constants *a* and *b*, find the derivative of: $\frac{x-a}{x-b}$.

Q65. For some constants *a* and *b*, find the derivative of: $(ax^2 + b)^2$.

Q66. Differentiate the following function with respect to *x*:

$$\frac{x \tan x}{\sec x + \tan x}$$

Q67. Find the derivative of the following function: $(ax + b)^n (cx + d)^m$.

Q68. Find the derivative of: $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$.

Q69. Find the derivative of: $\frac{px^2+qx+r}{ax+b}$.

Q70. Find the derivative of: $\frac{ax+b}{px^2+qx+r}$.

Q71. Find the derivative of: $\frac{1}{ax^2+bx+c}$.

**Q72.** Find the derivative of: $(ax + b)(cx + d)^2$.

**Q73.** Find the derivative of: $(px + q)\left(\frac{r}{x} + s\right)$. $p, q, r, s \in$ constant.

**Q74.** Find the derivative of the following function: $(x^2 + 1)\cos x$.

**Q75.** Find the derivative of the following function: $x^4(5\sin x - 3\cos x)$.

**Q76.** Find the derivative of the following function: $\frac{\sin(x+a)}{\cos x}$.

**Q77.** Find the derivative of the following function: $\frac{a + b\sin x}{c + d\cos x}$.

**Q78.** Find the derivative of the following function: $\frac{\sec x - 1}{\sec x + 1}$.

**Q79.** Find the derivative of the following function: $(x + \cos x)(x - \tan x)$.

**Q80.** Find the derivative of the following function: $(ax^2 + \sin x)(p + q\cos x)$.

**Q81.** Find the derivative of the following function: $\frac{x^2\cos(\pi/4)}{\sin x}$.

**Q82.** Find the derivative of the following function: $\frac{4x + 5\sin x}{3x + 7\cos x}$.

**Q83.** Find $\frac{d}{dx}\left(\frac{\cos x}{1 + \sin x}\right)$.

**Q84.** Find the derivative of $\frac{\sin x + \cos x}{\sin x - \cos x}$.

**Q85.** If $y = \frac{x}{\sin^n x}$. Find $\frac{dy}{dx}$.

**Q86.** Differentiate the following function with respect to $x$:

$$\frac{a + \sin x}{1 + a\sin x}$$

**Q87.** Differentiate the following function with respect to $x$:

$$\frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$$

**Q88.** Differentiate the following function with respect to $x$:

$$\frac{ax^2 + bx + c}{px^2 + qx + r}$$

**Q89.** Prove that $xy' = y(1-y)$. If $y = \frac{x}{x+5}$

Date: 12/10/2021

**S1.** $y = (x + a)^n$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}(x+a)^n$

$= n(x+a)^{n-1} \cdot \frac{d}{dx}(x+a)$

$= n(x+a)^{n-1}.$

**S2.** $y = \frac{x+a}{x+b}$

$\frac{dy}{dx} = \frac{(x+b)\cdot \frac{d}{dx}(x+a) - (x+a)\cdot \frac{d}{dx}(x+b)}{(x+b)^2}$

$= \frac{(x+b)-(x+a)}{(x+b)^2}$

$= \frac{1}{(x+b)} - \frac{(x+a)}{(x+b)^2}.$

**S3.** $y = \sin(x + a)$

$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}(\sin(x+a))$

$= \cos(x+a) \cdot \frac{d}{dx}(x+a) = \cos(x+a)$

**S4.** $y = (ax + b)^n$

$\frac{dy}{dx} = \frac{d}{dx}(ax+b)^n$

$= n(ax+b)^{n-1} \cdot \frac{d}{dx}(ax+b)$

$= na(ax+b)^{n-1}.$

**S5.** $y = (x^2 + x + 1) \cdot \sin 2x$

$\therefore \quad \frac{dy}{dx} = \left(\frac{d}{dx}(x^2+x+1)\right) \cdot \sin 2x + (x^2+x+1)\left(\frac{d}{dx}(\sin 2x)\right)$

$= (2x + 1) \cdot \sin 2x + 2(x^2 + x + 1) \cos 2x.$

**S6.** $\because$ $y = (x + 1)(x + 2)$

$\therefore$ $\frac{dy}{dx} = (x+1)\cdot\frac{d}{dx}(x+2) + (x+2)\cdot\frac{d}{dx}(x+1)$

$= (x + 1) + (x + 2) = 2x + 3.$

**S7.** $\because$ $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$\Rightarrow$ $y = x + \frac{1}{x} + 2$

$\frac{dy}{dx} = 1 - \frac{1}{x^2}.$

**S8.** $\because$ $\frac{d}{dx}\left(x + \frac{1}{x}\right)^2 = \frac{d}{dx}(x)^2 + \frac{d}{dx}\left(\frac{1}{x^2}\right) + \frac{d}{dx}(2)$

$\because$ $\frac{dy}{dx}$ (c) = 0, $c$ is constant

$= 2x - \frac{2}{x^3}.$

**S9.** $\because$ $y = (2 + x)^2$

$\therefore$ $\frac{dy}{dx} = \frac{d}{dx}(2+x)^2$

$= 2(2+x)\cdot\frac{d}{dx}(2+x)$

$= 2(2+x).$

**S10.** $\because$ $\frac{d}{dx}(x^n) = nx^{n-1},$

$\therefore$ $y = 5x^2$

$\therefore$ $\frac{dy}{dx} = 5\cdot\frac{d}{dx}(x^2)$

$\therefore$ $= 5 \times 2x = 10\,x.$

**S11.** Given, $y = 5 \cos x,$

$\frac{dy}{dx} = 5\cdot\frac{d}{dx}(\cos x)$

$\because \quad \frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx} f(x)$

$\therefore \quad \frac{dy}{dx} = -5 \sin x.$

**S12.** Recall the trigonometric formula $\sin 2x = 2 \sin x \cos x$. Thus

$$\frac{df(x)}{dx} = \frac{d}{dx}(2 \sin x \cos x) = 2\frac{d}{dx}(\sin x \cos x)$$

$$= 2 [(\sin x)' \cos x + \sin x (\cos x)']$$

$$= 2 [(\cos x) \cos x + \sin x (-\sin x)]$$

$$= 2 (\cos^2 x - \sin^2 x).$$

**S13.**

$$\frac{df(x)}{dx} = \frac{d}{dx}(\sin x \sin x)$$

$$= (\sin x)' \sin x + \sin x (\sin x)'$$

$$= (\cos x) \sin x + \sin x (\cos x)$$

$$= 2 \sin x \cos x = \sin 2x.$$

**S14.** Clearly, this function is defined every where except at $x = 0$. We use the quotient rule with $u = x + 1$ and $v = x$. Hence $u' = 1$ and $v' = 1$. Therefore

$$\frac{df(x)}{dx} = \frac{d}{dx}\left(\frac{x+1}{x}\right) = \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2} = \frac{1(x) - (x+1)1}{x^2} = -\frac{1}{x^2}.$$

**S15.** Let, $\sin x = p$ $\quad f(x) = p^n \quad \Rightarrow \quad f'(x) = np^{n-1} \cdot \frac{dp}{dx}$ $\quad$ **($P \in \boldsymbol{R}$)**

$\therefore \quad f'(x) = n \cdot \sin^{n-1} x \cdot \cos x.$

**S16.**

$$y = x^2 \tan x$$

$$\therefore \quad \frac{dy}{dx} = x^2 \cdot \frac{d}{dx} \cdot \tan x + \tan x \cdot \frac{d}{dx}(x^2)$$

$$= x^2 \cdot \sec x \cdot \tan x + 2x \cdot \tan x$$

$$= x \tan x [x \sec x + 2].$$

**S17.**

$$y = x^2 \sin x$$

$$\therefore \quad \frac{dy}{dx} = x^2 \cdot \frac{d}{dx} \cdot \sin x + \sin x \cdot \frac{d}{dx}(x^2)$$

$$= x^2 \cdot \cos x + 2x \cdot \sin x$$

$$= x (x \cos x + 2 \sin x).$$

**S18.**

$$y = (x + a) \cdot \tan 2x$$

$$\therefore \quad \frac{dy}{dx} = (x+a) \cdot \frac{d}{dx}(\tan 2x) + \tan 2x \cdot \frac{d}{dx}(x+a)$$

$$= (x + a)(2 \sec 2x \cdot \tan 2x) + \tan 2x.$$

**S19.** $y = \tan x^2$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{d}{dx}(\tan x^2) = \sec x^2 \cdot \tan x^2 \cdot \dfrac{d}{dx}(x^2)$

$= (\sec x^2 \cdot \tan x^2) \cdot 2x$

$= 2x \sec x^2 \cdot \tan x^2.$

**S20.** $y = x^2 \sin 3x$

$\therefore \quad \dfrac{dy}{dx} = x^2 \cdot \dfrac{d}{dx}(\sin 3x) + \sin 3x \cdot \dfrac{d}{dx}(x^2)$

$= 3x^2 \cdot (\cos 3x) + 2x \cdot \sin 3x$

$= x[3x \cos 3x + 2 \sin 3x].$

**S21.** $y = x^5 \cot x$

$\therefore \quad \dfrac{dy}{dx} = \cot x \cdot \dfrac{d}{dx}(x^5) + x^5 \cdot \dfrac{d}{dx}(\cot x)$

$= (\cot x) \cdot 5x^4 + x^5 \cdot (-\operatorname{cosec}^2 x)$

$= 5x^4 \cdot \cot x - x^5 \operatorname{cosec}^2 x.$

**S22.** By definition, $g(x) = \cot x$

We use the quotient rule on this function wherever it is defined.

$$\frac{dg}{dx} = \frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{(\cos x)'(\sin x) - (\cos x)(\sin x)'}{(\sin x)^2}$$

$$= \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$$

$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\operatorname{cosec}^2 x$$

Alternate method: $\dfrac{dg}{dx} = \dfrac{d}{dx}(\cot x) = \dfrac{d}{dx}\left(\dfrac{1}{\tan x}\right)$

$$= \frac{(1)'(\tan x) - (1)(\tan x)'}{(\tan x)^2}$$

$$= \frac{(0)(\tan x) - (\sec x)^2}{(\tan x)^2}$$

$$= \frac{-\sec^2 x}{\tan^2 x} = -\operatorname{cosec}^2 x$$

**S23.** Let $$f(x) = 5 \sin x - 6 \cos x + 7$$

$$f'(x) = \frac{d(5 \sin x - 6 \cos x + 7)}{dx}$$

$$= \frac{d(5 \sin x)}{dx} - \frac{d(6 \cos x)}{dx} + \frac{d(7)}{dx}$$

$$= 5 \cos x - 6(-\sin x) + 0$$

$$= 5 \cos x + 6 \sin x.$$

**S24.** Let $$f(x) = 5 \sec x + 4 \cos x$$

$$f'(x) = \frac{d(5 \sec x + 4 \cos x)}{dx}$$

$$= \frac{d(5 \sec x)}{dx} + \frac{d(4 \cos x)}{dx}$$

$$= 5\frac{d(\sec x)}{dx} + 4\frac{d(\cos x)}{dx}$$

$$= 5 \sec x \tan x - 4 \sin x.$$

**S25.** Let $$y = \operatorname{cosec} x \cdot \cot x$$

Hence, $$\frac{dy}{dx} = \cot x(-\operatorname{cosec} x \cot x) + \operatorname{cosec} x(-\operatorname{cosec}^2 x)$$

[Applying product rule]

$$= -\operatorname{cosec} x \cot^2 x - \operatorname{cosec}^3 x$$

$$= -\operatorname{cosec} x(\cot^2 x + \operatorname{cosec}^2 x).$$

**S26.** Let $$y = \sin(x + a)$$

$$\frac{dy}{dx} = \frac{d \sin(x+a)}{dx} = \cos(x+a) \cdot 1$$

$$= \cos(x + a).$$

**S27.** Let $$y = 4\sqrt{x} - 2$$

$$\frac{dy}{dx} = \frac{d(4\sqrt{x} - 2)}{dx} = \frac{d(4\sqrt{x})}{dx} - \frac{d(2)}{dx}$$

$$= \frac{4d(x^{1/2})}{dx} - 0$$

$$= 4\left(\frac{1}{2}\right)x^{-1/2} = \frac{2}{\sqrt{x}}.$$

**S28.** Let $$y = (x + a)$$

$$\frac{dy}{dx} = \frac{d(x+a)}{dx}$$

$$= \frac{d(x)}{dx} + \frac{d(a)}{dx} = 1 + 0 = 1.$$

**S29.** Let $$h(x) = \frac{x^5 - \cos x}{\sin x}$$

We use the quotient rule on this function wherever it is defined.

$$h'(x) = \frac{(x^5 - \cos x)' \sin x - (x^5 - \cos x)(\sin x)'}{(\sin x)^2}$$

$$= \frac{(5x^4 + \sin x)\sin x - (x^5 - \cos x)\cos x}{\sin^2 x}$$

$$= \frac{-x^5 \cos x + 5x^4 \sin x + 1}{(\sin x)^2}.$$

$$= \frac{5x^4 \sin x - x^5 \cos x + 1}{\sin^2 x}.$$

**S30.** Let $$f(x) = \operatorname{cosec} x$$

$$f'(x) = \frac{d(\operatorname{cosec} x)}{dx} = \frac{d\,\frac{1}{\sin x}}{dx}$$

$$= \frac{\sin x \frac{d(1)}{dx} - \frac{1 \cdot (\sin x)}{dx}}{(\sin x)^2}$$

$$= \frac{0 - \cos x}{\sin^2 x} = \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= -\operatorname{cosec} x \cot x.$$

**S31.** Let $$f(x) = x^5(3 - 6x^{-9}) = 3x^5 - 6x^{-4}$$

$$f'(x) = \frac{d(3x^5 - 6x^{-4})}{dx}$$

$$= \frac{d(3x^5)}{dx} - \frac{d(6x^{-4})}{dx}$$

$$= 3\frac{d(x^5)}{dx} - 6\frac{d(x^{-4})}{dx}$$

$$= (3)(5)x^4 - (6)(-4)x^{-5}$$

$$= 15x^4 + 24x^{-5}.$$

**S32.** Let $$f(x) = \sec x$$

$$f'(x) = \frac{d \sec x}{dx} = \frac{d\,\frac{1}{\cos x}}{dx}$$

$$= \frac{\cos x \frac{d(1)}{dx} - 1\frac{d \cos x}{dx}}{(\cos x)^2}$$

$$= \frac{0-(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos^2 x}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \sec x \tan x\,.$$

**S33.** Let $f(x) = \sin x \cos x$

Using product rule, we have

$$f'(x) = \sin x \frac{d \cos x}{dx} + \cos x \frac{d \sin x}{dx}$$

$$= \sin x\,(-\sin x) + \cos x\,(\cos x)$$

$$= -\sin^2 x + \cos^2 x = \cos^2 x - \sin^2 x.$$

**S34.** Let $f(x) = 2x - \frac{3}{4}$

$$f'(x) = \frac{d\left(2x - \frac{3}{4}\right)}{dx} = \frac{d\,(2x)}{dx} - \frac{d\left(\frac{3}{4}\right)}{dx}$$

$$= \frac{2dx}{dx} - 0 = 2.$$

**S35.** Let $f(x) = (x-a)(x-b)$

Using product rule, we have

$$\frac{df(x)}{dx} = (x-a)\frac{d\,(x-b)}{dx} + (x-b)\frac{d\,(x-a)}{dx}$$

$$= (x-a)\left[\frac{d\,(x)}{dx} - \frac{d\,(b)}{dx}\right] + (x-b)\left[\frac{d\,(x)}{dx} - \frac{d\,(a)}{dx}\right]$$

$$= (x-a)[1-0] + (x-b)[1-0]$$

$$= (x-a) + (x-b) = 2x - (a+b).$$

**S36.** We use the quotient rule on this function $\frac{x+\cos x}{\tan x}$ wherever it is defined.

$$h'(x) = \frac{(x+\cos x)' \tan x - (x+\cos x)(\tan x)'}{(\tan x)^2}$$

$$= \frac{(1-\sin x)\tan x - (x+\cos x)\sec^2 x}{(\tan x)^2}.$$

**S37.**

$$y = (ax)^m + b^m$$

$$y = a^m \cdot x^m + b^m$$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}(a^m \cdot x^m) + \frac{d}{dx}(b^m)$$

$$= ma^m \cdot x^{m-1}.$$

**S38.** $$y = 5 \cdot (\sin (1 + nx)),$$

$$\frac{dy}{dx} = 5 \cdot \frac{d}{dx}(\sin(1+nx))$$

$$= 5\cos(1+nx)\cdot\frac{d}{dx}(1+nx)$$

$$= 5\cdot\cos(1+nx)\cdot n = 5n\cdot\cos(1+nx).$$

**S39.** $\because$ $$y = x^n \tan x$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^n)\cdot\tan x + \frac{d}{dx}(\tan x)\cdot x^n$$
$$= nx^{n-1}\tan x - \sec^2 x\cdot x^n.$$

**S40.** $$y = \frac{x}{\cos x}$$

$\therefore$ $$\frac{dy}{dx} = \frac{\cos x\cdot\frac{d}{dx}(x) - x\cdot\frac{d}{dx}(\cos x)}{(\cos x)^2}$$

$$= \frac{\cos x - x\sin x}{\cos^2 x}.$$

**S41.** $$y = \frac{x}{1+\cos x}$$

$\therefore$ $$\frac{dy}{dx} = \frac{(1+\cos x)\cdot\frac{d}{dx}(x) - x\cdot\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x) - x(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{1+\cos x + x\sin x}{(1+\cos x)^2}.$$

**S42.** $$y = \frac{1}{1+\sin x}$$

$\therefore$ $$\frac{dy}{dx} = \frac{(1+\sin x)\cdot\frac{d}{dx}(1) - 1\cdot\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{\cos x}{(1+\sin x)^2}.$$

**S43.** $y = (x^2 - 5x + 6)(x^3 + 2)$

$\therefore$ $$\frac{dy}{dx} = (x^2 - 5x + 6)\cdot\frac{d}{dx}(x^3 + 2) + (x^3 + 2)\cdot\frac{d}{dx}(x^2 - 5x + 6)$$

$$= (x^2 - 5x + 6)\cdot 3x^2 + (x^3 + 2)(2x - 5)$$

$$= 5x^4 - 20x^3 + 18x^2 + 4x^2 + 4x - 10.$$

**S44.** $y = (1 - \cos x)(1 + \tan x)$

$$\frac{dy}{dx} = (1 - \cos x)\cdot\frac{d}{dx}(1 + \tan x) + (1 + \tan x)\cdot\frac{d}{dx}(1 - \cos x)$$

$$= (1 - \cos x)\cdot \sec^2 x + (1 + \tan x)\sin x$$

$$= \sec^2 x - \sec x + \sin x + \sin x \cdot \tan x.$$

**S45.** $y = (x^2 - 5x + 6)\cdot \sec x$

$\therefore$ $$\frac{dy}{dx} = \sec x\left\{\frac{d}{dx}(x^2 - 5x + 6)\right\} + \left\{\frac{d}{dx}(\sec x)\right\}\cdot(x^2 - 5x + 6)$$

$$= (2x - 5)\cdot\sec x + \sec x\cdot\tan x\,(x^2 - 5x + 6)$$

$$= \sec x\,[(x^2 - 5x + 6)\tan x + (2x - 5)].$$

**S46.** $y = (1 + 2\tan x)(5 + 4\cos x)$

$\therefore$ $$\frac{dy}{dx} = \left\{\frac{d}{dx}(1 + 2\tan x)\right\}\cdot(5 + 4\cos x) + (1 + 2\tan x)\cdot\left\{\frac{d}{dx}(5 + 4\cos x)\right\}$$

$$= 2\sec^2 x\,(5 + 4\cos x) + (1 + 2\tan x)(-4\sin x)$$

$$= -4\sin x + 10\sec^2 x + 8\cos x.$$

**S47.** $y = \dfrac{ax + b}{cx + d}$

$\therefore$ $$\frac{dy}{dx} = \frac{(cx + d)\cdot\frac{d}{dx}(ax + b) - (ax + b)\cdot\frac{d}{dx}(cx + d)}{(cx + d)^2}$$

$$= \frac{(cx + d)\cdot a - (ax + b)\cdot c}{(cx + d)^2}$$

$$= \frac{ad - bc}{(cx + d)^2}.$$

**S48.** $y = x^4(\sin x - \cos x)$

$\therefore$ $$\frac{dy}{dx} = (\sin x - \cos x)\cdot\frac{d}{dx}(x^4) - x^4\cdot\frac{d}{dx}(\sin x - \cos x)$$

$$= (\sin x - \cos x)\ 4x^3 - x^4 (\cos x + \sin x)$$

$$= (\sin x)(4x^3 - x^4) - \cos x\,(4x^3 + x^4).$$

**S49.** $\because$ $y = \cot x$

$\Rightarrow$ $y = \dfrac{\cos x}{\sin x}$

$\therefore$ $\dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{\cos x}{\sin x}\right)$

$$= \frac{\frac{d}{dx}(\cos x)\cdot \sin x - \cos x \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{(-\sin x)(\sin x) - \cos x \cdot \cos x}{(\sin x)^2}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\text{cosec}^2 x\,.$$

**S50.** $f(x) = \dfrac{x^n - a^n}{(x-a)}$

$\therefore$ $f'(x) = \dfrac{(x-a)\cdot\left\{\frac{d}{dx}x^n - \frac{d}{dx}a^n\right\} - (x^n - a^n)\left\{\frac{d}{dx}x - \frac{d}{dx}a\right\}}{(x-a)^2}$

$$= \frac{(x-a)(nx^{n-1}) - (x^n - a^n)\cdot 1}{(x-a)^2}$$

$$= \frac{nx^n - nax^{n-1} - x^n + a^n}{(x-a)^2}$$

$$= \frac{(n-1)x^n - a(nx^{n-1} - a^{n-1})}{(x-a)^2}\,.$$

**S51.** $f(x) = \dfrac{1+\frac{1}{x}}{1-\frac{1}{x}} = \dfrac{\frac{x+1}{x}}{\frac{x-1}{x}}$

$f(x) = \dfrac{x+1}{x-1}$

$$\therefore \quad f'(x) = \frac{\left[\frac{d}{dx}(x+1)\right]\cdot(x-1)-(x+1)\cdot\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2} = \frac{-2}{(x-1)^2}.$$

**S52.** Let $$f(x) = \frac{(x-1)(x-2)}{(x-3)(x-4)}$$

$$f'(x) = \frac{(x^2-7x+12)\frac{d}{dx}(x^2-3x+2)-(x^2-3x+2)\ \frac{d}{dx}(x^2-7x+12)}{(x^2-7x+12)^2}$$

$$= \frac{(x^2-7x+12)(2x-2)-(x^2-3x+2)(2x-7)}{[(x-3)\ (x-4)]^2}$$

$$= \frac{2x^3-14x^2+24x-3x^2+21x-36-2x^3\ +6x^2-4x+7x^2-21x+14}{(x-3)^2(x-4)^2}$$

$$= \frac{-4x^2+20x-22}{(x-3)^2(x-4)^2}.$$

**S53.** Let $$f(x) = x^n + ax^{n-1} + a^2x^{n-2} + ... + a^{n-1}x + a^n$$

$$\Rightarrow \quad f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + ... + a^{n-1}$$

$$\Rightarrow \quad f'(a) = nx^{n-1} + a(n-1)a^{n-2} + a^2(n-2)a^{n-3} + ... + a^{n-1}$$

$$= nx^{n-1} + (n-1)a^{n-1} + (n-2)a^{n-1} + ... + a^{n-1}$$

$$= a^{n-1}[n + (n-1) + (n-2) + ... + 1]$$

$$= a^{n-1}\left[\frac{n(n+1)}{2}\right] \qquad \left[\because \text{ Sum of } n \text{ natural number} = \frac{n(n+1)}{2}\right]$$

**S54.** $$y = \frac{x^2+3x-9}{x^2-9x+3}$$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left\{\frac{x^2+3x-9}{x^2-9x+3}\right\}$$

$$= \frac{(x^2-9x+3)\cdot\frac{d}{dx}(x^2+3x-9)-(x^2+3x-9)\cdot\frac{d}{dx}(x^2-9x+3)}{(x^2-9x+3)^2}$$

$$= \frac{(x^2-9x+3)(2x+3)-(x^2+3x-9)(2x-9)}{(x^2-9x+3)^2}.$$

**S55.**

$$y = \frac{x^2}{\sin x}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\sin x \cdot \frac{d}{dx}(x^2) - x^2 \cdot \frac{d}{dx}(\sin x)}{(\sin x)^2}$$

$$= \frac{2x \sin x - x^2 \cos x}{\sin^2 x}$$

$$= \frac{x(2 \sin x - x \cos x)}{\sin^2 x}.$$

**S56.**

$$y = \frac{x}{1 + \cot x}$$

$$\therefore \quad \frac{dy}{dx} = \frac{(1 + \cot x) \cdot \frac{d}{dx}(x) - x \cdot \frac{d}{dx}(1 + \cot x)}{(1 + \cot x)^2}$$

$$= \frac{(1 + \cot x) - x \cdot (-\text{cosec}^2 x)}{(1 + \cot x)^2}$$

$$= \frac{1 + x\,\text{cosec}^2\, x + \cot x)}{(1 + \cot x)^2}.$$

**S57.**

$$y = \frac{x}{1 + \tan x}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\left[\frac{d}{dx}(x)\right] \cdot (1 + \tan x) - \left[\frac{d}{dx}(1 + \tan x)\right] \cdot x}{(1 + \tan x)^2}$$

$$= \frac{(1 + \tan x) - (\sec x \cdot \tan x) \cdot x}{(1 + \tan x)^2}$$

$$= \frac{\tan x(-x \sec x + 1) + 1}{(1 + \tan x)^2}.$$

**S58.** Let

$$f(x) = 2 \tan x - 7 \sec x$$

$$f'(x) = \frac{d(2 \tan x - 7 \sec x)}{dx}$$

$$= \frac{d(2\tan x)}{dx} - \frac{d(7\sec x)}{dx}$$

$$= 2\frac{d\,\frac{\sin x}{\cos x}}{dx} - 7\sec x\tan x$$

$$= 2\left[\frac{\cos x\,\frac{d(\sin x)}{dx} - \sin x\,\frac{d(\cos x)}{dx}}{(\cos x)^2}\right] - 7\sec x\tan x$$

$$= 2\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right) - 7\sec x\tan x$$

$$= 2\sec^2 x - 7\sec x\tan x.$$

**S59.** Let

$$f(x) = 3\cot x + 5\operatorname{cosec} x$$

$$f'(x) = \frac{d(3\cot x + 5\operatorname{cosec} x)}{dx}$$

$$= \frac{d(3\cot x)}{dx} + \frac{d(5\operatorname{cosec} x)}{dx}$$

$$= \frac{3d(\cot x)}{dx} + \frac{5d(\operatorname{cosec} x)}{dx}$$

$$= 3\left[\frac{d\,\frac{\cos x}{\sin x}}{dx}\right] + 5(-\operatorname{cosec} x\cot x)$$

$$= 3\left[\frac{\sin x\,\frac{d(\cos x)}{dx} - \cos x\,\frac{d(\sin x)}{dx}}{(\sin x)^2}\right] - 5\operatorname{cosec} x\cot x$$

$$= 3\left[\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}\right] - 5\operatorname{cosec} x\cot x$$

$$= 3\left[\frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x}\right] - 5\operatorname{cosec} x\cot x$$

$$= -3\operatorname{cosec}^2 x - 5\operatorname{cosec} x\cot x.$$

**S60.** Let

$$f(x) = \frac{2}{x+1} - \frac{x^2}{3x-1}$$

$$f'(x) = \frac{d\left[\frac{2}{x+1} - \frac{x^2}{3x-1}\right]}{dx}$$

$$= \frac{d\left[\frac{2}{x+1}\right]}{dx} - \frac{d\left[\frac{x^2}{3x-1}\right]}{dx}$$

$$= \frac{(x+1)\frac{d(2)}{dx} - 2\frac{d(x+1)}{dx}}{(x+1)^2} - \frac{(3x-1)\frac{d(x^2)}{dx} - x^2\frac{d(3x-1)}{dx}}{(3x-1)^2}$$

$$= \frac{0 - 2\left[\frac{dx}{dx} + \frac{d1}{dx}\right]}{(x+1)^2} - \frac{(3x-1)(2x) - x^2\left[\frac{d(3x)}{dx} - \frac{d(1)}{dx}\right]}{(3x-1)^2}$$

$$= \frac{0 - 2[1+0]}{(x+1)^2} - \frac{(3x-1)(2x) - x^2[(3) - 0]}{(3x-1)^2}$$

$$= \frac{-2}{(x+1)^2} - \frac{(6x^2 - 2x) - 3x^2}{(3x-1)^2}$$

$$= \frac{-2}{(x+1)^2} - \frac{3x^2 - 2x}{(3x-1)^2}.$$

**S61.** Let $f(x) = x^{-4}(3 - 4x^{-5})$

$$f'(x) = x^{-4}\frac{d(3-4x^{-5})}{dx} + (3-4x^{-5})\frac{dx^{-4}}{dx}$$

$$= x^{-4}\left[\frac{d(3)}{dx} - \frac{d(4x^{-5})}{dx}\right] + (3-4x^{-5})(-4)x^{-5}$$

$$= x^{-4}[0 - (4)(-5)x^{-6}] + (3 - 4x^{-5})(-4)x^{-5}$$

$$= x^{-4}[20x^{-6}] - x^{-5}(12 - 16x^{-5})$$

$$= 20x^{-10} - 12x^{-5} + 16x^{-10} = 36x^{-10} - 12x^{-5}.$$

**Another method:**

Let $f(x) = x^{-4}(3 - 4x^{-5}) = 3x^{-4} - 4x^{-9}$

Hence, $f'(x) = \frac{d(3x^{-4} - 4x^{-9})}{dx}$

$$= \frac{d(3x^{-4})}{dx} - \frac{d(4x^{-9})}{dx}$$

$$= (3)(-4)x^{-5} - (4)(-9)x^{-10}$$

$$= -12x^{-5} + 36x^{-10}$$

$$= 36x^{-10} - 12x^{-5}.$$

**S62.** Let $f(x) = x^{-3}(5 + 3x)$

$$f'(x) = \frac{dx^{-3}(5+3x)}{dx}$$

$$= x^{-3}\frac{d(5+3x)}{dx}+(5+3x)\frac{dx^{-3}}{dx}$$

$$= x^{-3}\left[\frac{d(5)}{dx}+\frac{d(3x)}{dx}\right]+(5+3x)(-3)x^{-3-1}$$

$$= x^{-3}\left[0+3\frac{dx}{dx}\right]+(5+3x)(-3)x^{-4}$$

$$= 3x^{-3} - (15 + 9x)x^{-4}$$

$$= \frac{3}{x^3}-\frac{(15+9x)}{x^4}=\frac{3x-15-9x}{x^4}$$

$$= \frac{-6x-15}{x^4}=\frac{-3(2x+5)}{x^4}$$

**S63.** Let

$$f(x) = (5x^3 + 3x - 1)(x - 1)$$

$$f'(x) = (x-1)\frac{d(5x^3+3x-1)}{dx}+(5x^3+3x-1)\frac{d(x-1)}{dx}$$

$$= (x-1)\left[\frac{d}{dx}(5x^3)+\frac{d(3x)}{dx}-\frac{d(1)}{dx}\right]+(5x^3+3x-1)\left[\frac{d(x)}{dx}-\frac{d(1)}{dx}\right]$$

$$= (x-1)\left[5\frac{d(x^3)}{dx}+\frac{3\,dx}{dx}-0\right]+(5x^3+3x-1)[1-0]$$

$$= (x - 1)[5(3x^{3-1}) + 3] + (5x^3 + 3x - 1)$$

$$= (x - 1)[15x^2 + 3] + (5x^3 + 3x - 1)$$

$$= 15x^3 + 3x - 15x^2 - 3 + 5x^3 + 3x - 1)$$

$$= 20x^3 - 15x^2 + 6x - 4.$$

**S64.** Let

$$f(x) = \frac{x-a}{x-b}$$

Using product rule, we have

$$= \frac{(x-b)\frac{d(x-a)}{dx}-(x-a)\frac{d(x-b)}{dx}}{(x-b)^2}$$

$$= \frac{(x-b)\left[\frac{d(x)}{dx}-\frac{d(a)}{dx}\right]-(x-a)\left[\frac{d(x)}{dx}-\frac{d(b)}{dx}\right]}{(x-b)^2}$$

$$= \frac{(x-b)[1-0]-(x-a)[1-0]}{(x-b)^2}$$

$$= \frac{(x-b)-(x-a)}{(x-b)^2}$$

$$= \frac{x-b-x+a}{(x-b)^2}=\frac{a-b}{(x-b)^2}.$$

**S65.** Let
$$f(x) = (ax^2 + b)^2$$
$$\frac{df(x)}{dx} = \frac{d}{dx}(ax^2 + b)\cdot(ax^2 + b)$$

Using product rule, we have

$$= (ax^2 + b)\frac{d(ax^2 + b)}{dx} + (ax^2 + b)\frac{d(ax^2 + b)}{dx}$$
$$= (ax^2 + b)\left[\frac{d}{dx}(ax^2) + \frac{d(b)}{dx}\right] + (ax^2 + b)\left[\frac{d}{dx}(ax^2) + \frac{d(b)}{dx}\right]$$
$$= (ax^2 + b)\left[a\frac{d(x^2)}{dx} + 0\right] + (ax^2 + b)\left[a\frac{d}{dx}(x^2) + 0\right]$$
$$= (ax^2 + b)\,[a(2x^{2-1})] + (ax^2 + b)\,[a(2x^{2-1})]$$
$$= (ax^2 + b)\,[2ax] + (ax^2 + b)\,[2ax]$$
$$= 4ax\,(ax^2 + b).$$

**S66.** Let $f(x) = \dfrac{x\tan x}{\sec x + \tan x}$

$$f'(x) = \frac{(\sec x + \tan x)\frac{d}{dx}(x\tan x) - (x\tan x)\frac{d}{dx}(\sec x + \tan x)}{(\sec x + \tan x)^2}$$
$$= \frac{(\sec x + \tan x)\left(x\frac{d}{dx}\tan x + \tan x\frac{d}{dx}(x)\right) - (x\tan x)[\sec x\tan x + \sec^2 x]}{(\sec x + \tan x)^2}$$
$$= \frac{(\sec x + \tan x)(x\sec^2 x + \tan x) - (x\tan x)(\sec x)[\tan x + \sec x]}{(\sec x + \tan x)^2}$$
$$= \frac{(\sec x + \tan x)(x\sec^2 x + \tan x - x\sec x\tan x]}{(\sec x + \tan x)^2} = \frac{x\sec x[\sec x - \tan x] + \tan x}{(\sec x + \tan x)^2}.$$

**S67.** Let
$$y = (ax + b)^n (cx + d)^m$$
$$\frac{dy}{dx} = (ax + b)^n\frac{d(cx + d)^m}{dx} + (cx + d)^m\frac{d(ax + b)^n}{dx}$$
$$= (ax + b)^n\cdot m(cx + d)^{m-1}\cdot c + (cx + d)^m\cdot n(ax + b)^{n-1}\cdot a$$
$$= mc(ax + b)^n(cx + d)^{m-1} + an(cx + d)^m(ax + b)^{n-1}$$
$$= (ax + b)^n(cx + d)^m\left[\frac{mc}{(cx + d)} + \frac{an}{(ax + b)}\right].$$

**S68.** Let

$$y = \frac{a}{x^4} - \frac{b}{x^2} + \cos x$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{a}{x^4} - \frac{b}{x^2} + \cos x\right)$$

$$= a \cdot \frac{d\left(\frac{1}{x^4}\right)}{dx} - b \cdot \frac{d\left(\frac{1}{x^2}\right)}{dx} + \frac{d(\cos x)}{dx}$$

$$= a \cdot (-4)(x^{-5}) - b \cdot (-2)(x^{-3}) - \sin x$$

$$= \frac{-4a}{x^5} + \frac{2b}{x^3} - \sin x.$$

**S69.** Let

$$y = \frac{px^2 + qx + r}{ax + b}$$

$$\frac{dy}{dx} = \frac{(ax+b)\frac{d(px^2+qx+r)}{dx} - (px^2+qx+r)\frac{d(ax+b)}{dx}}{(ax+b)^2}$$

$$= \frac{2apx^2 + aqx + 2bpx + bq - (apx^2 + aqx + ar)}{(ax+b)^2}$$

$$= \frac{apx^2 + 2bpx + bq - ar}{(ax+b)^2}.$$

**S70.** Let

$$y = \frac{ax + b}{px^2 + qx + r}$$

$$\frac{dy}{dx} = \frac{(px^2+qx+r)\frac{d(ax+b)}{dx} - (ax+b)\frac{d(px^2+qx+r)}{dx}}{(px^2+qx+r)^2}$$

$$= \frac{(px^2+qx+r)\cdot(a) - (ax+b)(2px+q)}{(px^2+qx+r)^2}$$

$$= \frac{apx^2 + aqx + ar - 2apx^2 - aqx - 2bpx - bq}{(px^2+qx+r)^2}$$

$$= \frac{-apx^2 - 2bpx + ar - bq}{(px^2+qx+r)^2}.$$

**S71.** Let

$$y = \frac{1}{ax^2 + bx + c}$$

$$\frac{dy}{dx} = \frac{(ax^2+bx+c)\frac{d(1)}{dx} - 1 \cdot \frac{d(ax^2+bx+c)}{dx}}{(ax^2+bx+c)^2}$$

$$= \frac{0 - a(2x) - b}{(ax^2 + bx + c)^2} = \frac{-b - 2ax}{(ax^2 + bx + c)^2}$$

$$= \frac{-(b + 2ax)}{(ax^2 + bx + c)^2}.$$

**S72.** Let $y = (ax + b)(cx + d)^2$

$$\frac{dy}{dx} = (ax + b)\frac{d(cx + d)^2}{dx} + (cx + d)^2 \frac{d(ax + b)}{dx}$$

$$= (ax + b)\left[(cx + d)\frac{d(cx + d)}{dx} + (cx + d)\frac{d(cx + d)}{dx}\right] + (cx + d)^2 [a + 0]$$

$$= (ax + b)[(cx + d)(c) + (cx + d)(c)] + a(cx + d)^2$$

$$= (ax + b)\, 2c(cx + d) + a(cx + d)^2$$

$$= (cx + d)[2c(ax + b) + a(cx + d)].$$

**S73.** Let $y = (px + q)\left(\frac{r}{x} + s\right)$

$$\frac{dy}{dx} = (px + q)\frac{d\left(\frac{r}{x} + s\right)}{dx} + \left(\frac{r}{x} + s\right)\frac{d(px + q)}{dx}$$

$$= (px + q)\left[r\frac{d\left(\frac{1}{x}\right)}{dx} + \frac{d(s)}{dx}\right] + \left(\frac{r}{x} + s\right)\left[\frac{d(px)}{dx} + \frac{dq}{dx}\right]$$

$$= (px + q)\left[r\left(-\frac{1}{x^2}\right) + 0\right] + \left(\frac{r}{x} + s\right)[p + 0]$$

$$= -(px + q)\left(\frac{r}{x^2}\right) + p\left(\frac{r}{x} + s\right).$$

**S74.** Let $y = (x^2 + 1)\cos x$

$$\frac{dy}{dx} = \frac{d(x^2 + 1)\cos x}{dx}$$

$$= (x^2 + 1)\frac{d\cos x}{dx} + \cos x\,\frac{d(x^2 + 1)}{dx}$$

$$= (x^2 + 1)(-\sin x) + \cos x\,(2x)$$

$$= -(x^2 + 1)\sin x + 2x\cos x.$$

**S75.** Let $y = x^4(5\sin x - 3\cos x)$

$$\frac{dy}{dx} = \frac{x^4 d(5\sin x - 3\cos x)}{dx} + (5\sin x - 3\cos x)\frac{dx^4}{dx}$$

$$= x^4[5\cos x - 3(-\sin x)] + (5\sin x - 3\cos x)(4x^3)$$

$$= x^4(5\cos x + 3\sin x) + 4x^3(5\sin x - 3\cos x).$$

**S76.** Let

$$y = \frac{\sin(x+a)}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x \dfrac{d \sin(x+a)}{dx} - \sin(x+a)\dfrac{d \cos x}{dx}}{(\cos x)^2}$$

$$= \frac{\cos x \cos(x+a) - \sin(x+a)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos x \cos(x+a) + \sin(x+a)\sin x}{\cos^2 x}$$

$$= \frac{\cos(x+a-x)}{\cos^2 x} = \frac{\cos a}{\cos^2 x}.$$

**S77.** Let

$$y = \frac{a + b\sin x}{c + d\cos x}$$

$$\frac{dy}{dx} = \frac{(c + d\cos x)\dfrac{d(a + b\sin x)}{dx} - (a + b\sin x)\dfrac{d(c + d\cos x)}{dx}}{(c + d\cos x)^2}$$

$$= \frac{(c + d\cos x)(b\cos x) - (a + b\sin x)(-d\sin x)}{(c + d\cos x)^2}$$

$$= \frac{bc\cos x + bd\cos^2 x + ad\sin x + bd\sin^2 x}{(c + d\cos x)^2}$$

$$= \frac{ad\sin x + cb\cos x + bd}{(c + d\cos x)^2}.$$

**S78.**

Let

$$y = \frac{\sec x - 1}{\sec x + 1} = \frac{\dfrac{1}{\cos x} - 1}{\dfrac{1}{\cos x} + 1} = \frac{1 - \cos x}{1 + \cos x}$$

$$\frac{dy}{dx} = \frac{(1 + \cos x)\dfrac{d(1 - \cos x)}{dx} - (1 - \cos x)\dfrac{d(1 + \cos x)}{dx}}{(1 + \cos x)^2}$$

$$= \frac{(1 + \cos x)(\sin x) - (1 - \cos x)(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{\sin x + \sin x\cos x + \sin x - \sin x\cos x}{(1 + \cos x)^2}$$

$$= \frac{2\sin x}{(1 + \cos x)^2}.$$

**S79.** Let

$$y = (x + \cos x)(x - \tan x)$$

$$\frac{dy}{dx} = (x + \cos x)\frac{d(x - \tan x)}{dx} + (x - \tan x)\frac{d(x + \cos x)}{dx}$$

$$= (x + \cos x)(1 - \sec^2 x) + (x - \tan x)(1 - \sin x)$$

$$= (x + \cos x)(-\tan^2 x) + (x - \tan x)(1 - \sin x)$$

$$= -x\tan^2 x - \sin^2 x \sec x + x - x\sin x + \sin^2 x \sec x - \tan x$$

$$= x(1 - \sin x - \tan^2 x) - \tan x.$$

**S80.** Let
$$y = (ax^2 + \sin x)(p + q\cos x)$$

$$\frac{dy}{dx} = (ax^2 + \sin x)\frac{d(p + q\cos x)}{dx} + (p + q\cos x)\frac{d(ax^2 + \sin x)}{dx}$$

$$= (ax^2 + \sin x)\left[\frac{dp}{dx} + q\,\frac{d\cos x}{dx}\right] + (p + q\cos x)\left[\frac{dax^2}{dx} + \frac{d\sin x}{dx}\right]$$

$$= (ax^2 + \sin x)[0 + q(-\sin x)] + (p + q\cos x)[2ax + \cos x]$$

$$= -q\sin x\,(ax^2 + \sin x) + (p + q\cos x)(2ax + \cos x).$$

**S81.** Let
$$y = \frac{x^2\cos(\pi/4)}{\sin x} = \frac{x^2}{\sqrt{2}\sin x}$$

$$\frac{dy}{dx} = \frac{\sqrt{2}\sin x\,\dfrac{dx^2}{dx} - x^2\,\dfrac{d\sqrt{2}\sin x}{dx}}{(\sqrt{2}\sin x)^2}$$

$$= \frac{\sqrt{2}\sin x(2x) - x^2(\sqrt{2})\cos x}{2\sin^2 x}$$

$$= \frac{2\sqrt{2}\,x\sin x - \sqrt{2}\,x^2\cos x}{2\sin^2 x}$$

$$= \frac{\sqrt{2}\,.\,x(2\sin x - x\cos x)}{2\sin^2 x}.$$

**S82.** Let
$$y = \frac{4x + 5\sin x}{3x + 7\cos x}$$

$$\frac{dy}{dx} = \frac{(3x + 7\cos x)\dfrac{d(4x + 5\sin x)}{dx} - (4x + 5\sin x)\dfrac{d(3x + 7\cos x)}{dx}}{(3x + 7\cos x)^2}$$

$$= \frac{(3x + 7\cos x)(4 + 5\cos x) - (4x + 5\sin x)(3 - 7\sin x)}{(3x + 7\cos x)^2}$$

$$= \frac{(12x + 15x\cos x + 28\cos x + 35\cos^2 x) - (12x - 28x\sin x + 15\sin x - 35\sin^2 x)}{(3x + 7\cos x)^2}$$

$$= \frac{12x + 15x\cos x + 28\cos x + 35\cos^2 x - 12x + 28x\sin x - 15\sin x + 35\sin^2 x}{(3x + 7\cos x)^2}$$

$$= \frac{15x\cos x + 28\cos x + 28x\sin x - 15\sin x + 35}{(3x + 7\cos x)^2}.$$

**S83.**

$$\frac{d}{dx}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\left[\frac{d}{dx}(\cos x)\right]\cdot(1+\sin x)-\cos x\cdot\frac{d}{dx}(1+\sin x)}{(1+\sin x)^2}$$

$$= \frac{-\sin x\,(1+\sin x)-\cos x\cdot\cos x}{(1+\sin x)^2}$$

$$= \frac{-[\sin x+\sin^2 x+\cos^2 x]}{(1+\sin x)^2}$$

$$= \frac{-[\sin x+1]}{(1+\sin x)^2} = -\frac{1}{1+\sin x}.$$

**S84.**

$$f(x) = \frac{\sin x+\cos x}{\sin x-\cos x}$$

$$f'(x) = \frac{(\sin x-\cos x)\cdot\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\cdot\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$$

$$= \frac{(\sin x-\cos x)(\cos x-\sin x)-(\sin x+\cos x)(\cos x+\sin x)}{(\sin x-\cos x)^2}$$

$$= \frac{-2\sin^2 x-2\cos^2 x}{(\sin x-\cos x)^2} = \frac{-2(\sin^2 x+\cos^2 x)}{(\sin x-\cos x)^2}$$

$$= \frac{-2}{(\sin x-\cos x)^2}.$$

**S85.** $\because$ $$f(x) = \frac{x}{\sin^n x}$$

$\because$ $$\frac{d}{dx}(\sin^n x) = n\cdot\sin^{n-1}x\cdot\cos x$$

$\therefore$ $$f'(x) = \frac{\left[\frac{d}{dx}(x)\right]\cdot\sin^n x - x\cdot\frac{d}{dx}(\sin^n x)}{(\sin^n x)^2}$$

$$= \frac{1-\sin^n x - x\cdot n\cdot\sin^{n-1}x\cdot\cos x}{\sin^{2n}x}$$

$$= \frac{\sin^{n-1}x\,(\sin x - nx\cos x)}{\sin^{2n}x}$$

$$= \frac{\sin x - nx\cos x}{\sin^{n+1}x}.$$

**S86.** Let

$$f(x) = \frac{a + \sin x}{1 + a \sin x}$$

$$f'(x) = \frac{(1 + a \sin x)\frac{d}{dx}(a + \sin x) - (a + \sin x)\frac{d}{dx}(1 + a \sin x)}{(1 + a \sin x)^2}$$

$$= \frac{(1 + a \sin x)(0 + \cos x) - (a + \sin x)(0 + a \cos x)}{(1 + a \sin x)^2}$$

$$= \frac{(1 + a \sin x)(\cos x) - (a + \sin x)(a \cos x)}{(1 + a \sin x)^2}$$

$$= \frac{\cos x + a \sin x \cos x - a^2 \cos x - a \sin x \cos x}{(1 + a \sin x)^2}$$

$$= \frac{(1 - a^2)\cos x}{(1 + a \sin x)^2}.$$

**S87.** Let

$$f(x) = \frac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$$

$$f'(x) = \frac{(\sqrt{a} - \sqrt{x})\frac{d}{dx}(\sqrt{a} + \sqrt{x}) - (\sqrt{a} + \sqrt{x})\frac{d}{dx}(\sqrt{a} - \sqrt{x})}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{(\sqrt{a} - \sqrt{x})\left(0 + \frac{d}{dx}(x)^{\frac{1}{2}}\right) - (\sqrt{a} + \sqrt{x})\left(0 - \frac{d}{dx}(x^{\frac{1}{2}})\right)}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{(\sqrt{a} - \sqrt{x})\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - \left(\frac{1}{2}\right)(\sqrt{a} + \sqrt{x})x^{-\frac{1}{2}}}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{(\sqrt{a} - \sqrt{x})\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}}(\sqrt{a} + \sqrt{x})}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}[\sqrt{a} - \sqrt{x} + \sqrt{a} + \sqrt{x}]}{(\sqrt{a} - \sqrt{x})^2}$$

$$= \frac{1}{2\sqrt{x}} \times \frac{2\sqrt{a}}{(\sqrt{a} - \sqrt{x})^2} = \frac{\sqrt{a}}{\sqrt{x}(\sqrt{a} - \sqrt{x})^2}.$$

**S88.** Let $f(x) = \dfrac{ax^2 + bx + c}{px^2 + qx + r}$

$$f'(x) = \frac{(px^2 + qx + r)\,\dfrac{d}{dx}(ax^2 + bx + c) - (ax^2 + bx + c)\,\dfrac{d}{dx}(px^2 + qx + r)}{(px^2 + qx + r)^2}$$

$$= \frac{(px^2 + qx + r)(a \cdot 2x + b + 0) - (ax^2 + bx + c)(p \cdot 2x + q + 0)}{(px^2 + qx + r)^2}$$

$$= \frac{(px^2 + qx + r)(2ax + b) - (ax^2 + bx + c)(2px + q)}{(px^2 + qx + r)^2}$$

$$= \frac{(2apx^3 + 2aqx^2 + 2arx + pbx^2 + bqx + br) - (2apx^3 + 2pbx^2 + 2pcx + qax^2 + qbx + qc)}{(px^2 + qx + r)^2}$$

$$= \frac{2apx^3 + 2aqx^2 + pbx^2 + 2arx + bqx + br - 2apx^3 - 2pbx^2 - qax^2 - 2pcx - qbx - qc}{(px^2 + qx + r)^2}$$

$$= \frac{(aq - bp)\,x^2 + 2\,(ar - pc)\,x + br - qc}{(px^2 + qx + r)^2}.$$

**S89.** Let
$$y = \frac{x}{x+5} \qquad \text{... (i)}$$

$$\frac{dy}{dx} = \frac{(x+5)\,\dfrac{d}{dx}(x) - x\,\dfrac{d}{dx}(x+5)}{(x+5)^2}$$

$$\frac{dy}{dx} = \frac{(x+5)\cdot 1 - x \cdot (1+0)}{(x+5)^2} = \frac{x + 5 - x}{(x+5)^2} = \frac{5}{(x+5)^2}$$

Multiplying by $x$ on both sides

$$x\,\frac{dy}{dx} = \frac{5x}{(x+5)^2} = \frac{x}{(x+5)} \cdot \frac{5}{x+5}$$

$$x\,\frac{dy}{dx} = y \cdot (1 - y) \qquad \text{[By (i)]} \qquad \left[\begin{aligned} &\text{as } 1 - y = 1 - \frac{x}{x+5} \\ &1 - y = \frac{x + 5 - x}{x+5} = \frac{5}{x+5} \end{aligned}\right]$$

Date: 12/10/2021

**Q1.** Find the slope of tangent at $\theta = \frac{\pi}{2}$ of curve $y = \cot \theta$.

**Q2.** If $f(x) = x^2 - 9x + 20$, then find $f'(x)$ and hence find $f'(100)$.

**Q3.** Find the derivative of $x$ at $x = 1$.

**Q4.** Find the slope of tangent at point $x = 2$ of the curve $f(x) = x^3 - 3x^2 + x + 1$.

**Q5.** Find the slope of tangent at point $x = 1$ of the curve $y = (x + a)(x + b)$. Where $a$ and $b$ are unity.

**Q6.** Find the slope of tangent at $\theta = \frac{\pi}{2}$ of the curve $y = \sin \theta$

**Q7.** Find the slope of tangent at point $x = 1$ of the curve $y = (x^3 - 1)(x^2 + 1)$.

**Q8.** If the slope of tangent at $x = 1$ of the curve $y = x^2 + bx + 1$ is 3 unit. Find all real values of $b$.

**Q9.** If the slope of tangent at $x = 1$ of the curve $y = x^3 + bx$ is 12 unit. Find all possible values of $b$

**Q10.** Find the slope of tangent at $\theta = \frac{\pi}{3}$ of the curve $y = \frac{\theta}{\sin \theta}$.

**Q11.** Find the slope of tangent at $\theta = \frac{\pi}{4}$ of the curve $y = \frac{\theta}{\cos \theta}$.

**Q12.** If $y = \frac{x}{x+5}$. Prove that $x \cdot \frac{dy}{dx} = y(1 - y)$.

**Q13.** If $y = \theta \cdot \sin \theta$. Find slope of tangent at point $x = \frac{\pi}{2}$.

**Q14.** For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + 1$. Prove that $f'(1) = 100 \cdot f'(0)$.

Date: 12/10/2021

**S1.** Slope of tangent at point $x = x_0$ is given by $\left.\frac{dy}{dx}\right|_{x = x_0}$

$\therefore \quad y = \cot\theta$

$\therefore \quad \frac{dy}{dx} = -\text{cosec}^2\theta$

$\therefore \quad \left.\frac{dy}{dx}\right|_{x = x_0} = -1.$

**S2.** $\because \quad f(x) = x^2 - 9x + 20$

$\therefore \quad f'(x) = 2x - 9$

$\therefore \quad f'(100) = 2 \times 100 - 9 = 191.$

**S3.** Derivative of $f(x)$ at $x = 1$ is

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(1+h) - 1}{h} = \lim_{h \to 0} \frac{h}{h} = 1.$$

**S4.** $\because$ Slope of tangent at $x = x_0$ is being given by $\left.\frac{dy}{dx}\right|_{x = x_0}$

$\therefore \quad f(x) = x^3 - 3x^2 + x + 1$

$f'(x) = 3x^2 - 6x + 1$

$\therefore \quad \left.\frac{dy}{dx}\right|_{x = 2} = 3 \times 2^2 - 6 \times 2 + 1 = 1$

$\therefore$ Slope of tangent = 1.

**S5.** $\because \quad a = 1, \quad b = 1$ (given)

$\therefore \quad y = (x + 1)(x + 1)$

$y = (x^2 + 2x + 1)$

$\therefore \quad \dfrac{dy}{dx} = (2x+2)$

$\therefore \quad \left.\dfrac{dy}{dx}\right|_{x=1} = 4.$

**S6.**

$\because$ Slope of tangent at point $x = x_0$ is given by $\left.\dfrac{dy}{dx}\right|_{x=x_0}$

$\therefore \quad y = \sin\theta$

$\therefore \quad \dfrac{dy}{d\theta} = \cos\theta$

$\therefore \quad \left.\dfrac{dy}{d\theta}\right|_{\theta=\frac{\pi}{2}} = 0$

Hence at $\theta = \dfrac{\pi}{2}$, tangent is parallel to $x$-axis.

**S7.**

$\because$ Slope of tangent at $x = x_0$ is given by $\left.\dfrac{dy}{dx}\right|_{x=x_0}$

$\therefore \quad y = (x^3 - 1)(x^2 + 1)$

$$\frac{dy}{dx} = \left\{\frac{d}{dx}(x^3-1)\right\}\cdot(x^2+1) + \left\{\frac{d}{dx}(x^2+1)\right\}\cdot(x^3-1)$$

$$= (3x^2 - 1)(x^2 + 1) + 2x + 1(x^3 - 1)$$

$$= 3x^4 + 3x^2 + 2x^5 - 2x$$

$$= 2x^5 + 3x^4 + 3x^2 - 2x$$

$\therefore \quad \left.\dfrac{dy}{dx}\right|_{x=1} = 2 + 3 + 3 - 2 = 6.$

**S8.**

Slope of tangent at point $x = x_0$ is given by $\left.\dfrac{dy}{dx}\right|_{x=x_0}$

$y = x^2 + bx + 1$

$\therefore \quad \left.\dfrac{dy}{dx}\right|_{x=1} = 2x + b$

$\because \quad 2 + b = 3 \quad \Rightarrow \quad b = 1$

$\therefore \quad b = 1.$

**S9.**

$\because$ Slope of tangent at point $x = x_0$ is given by $\left.\frac{dy}{dx}\right|_{x = x_0}$

$\therefore$ $$y = x^3 + bx$$

$\therefore$ $$\frac{dy}{dx} = 3x^2 + b$$

$\therefore$ $$\left.\frac{dy}{dx}\right|_{x=1} = 3 + b$$

$\therefore$ $$3 + b = 12 \Rightarrow b = 9.$$

**S10.** Slope of tangent at point $x = x_0$ is given by $\left.\frac{dy}{dx}\right|_{x = x_0}$

$\therefore$ $$y = \frac{\theta}{\sin\theta}$$

$\therefore$ $$\frac{dy}{d\theta} = \frac{\sin\theta \cdot \frac{d\theta}{d\theta} - \theta \cdot \frac{d}{d\theta}(\sin\theta)}{(\sin\theta)^2}$$

$$= \frac{\sin\theta - \theta \cdot \cos\theta}{\sin^2\theta}$$

$\therefore$ $$\left.\frac{dy}{d\theta}\right|_{\theta = \frac{\pi}{3}} = \frac{\sin\frac{\pi}{3} - \frac{\pi}{3} \cdot \cos\frac{\pi}{3}}{\left(\sin^2\frac{\pi}{3}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2} - \frac{\pi}{3} \cdot \frac{1}{2}}{\frac{3}{4}} = \frac{\frac{\sqrt{3}}{2} - \frac{\pi}{6}}{\frac{3}{4}}$$

$$= \left(\frac{3\sqrt{3} - \pi}{\cancel{6}_3}\right) \times \frac{\cancel{4}^2}{3} = \frac{2}{9}\left(3\sqrt{3} - \pi\right).$$

**S11.** Slope of tangent at point $x = x_0$ is given by $\left.\frac{dy}{dx}\right|_{x = x_0}$

$\therefore$ $$y = \frac{\theta}{\cos\theta}$$

$\therefore$ $$\frac{dy}{d\theta} = \frac{\cos\theta \cdot \frac{d\theta}{d\theta} - \theta \cdot \frac{d}{d\theta}(\cos\theta)}{(\cos\theta)^2}$$

$$= \frac{\cos\theta + \theta\cdot\sin\theta}{\cos^2\theta}$$

$\therefore$ Slope of tangent at $\theta = \frac{\pi}{4}$

$$= \frac{\cos\frac{\pi}{4} + \frac{\pi}{4}\cdot\sin\frac{\pi}{4}}{\cos^2\frac{\pi}{4}}$$

$$= \frac{\frac{1}{\sqrt{2}} + \frac{\pi}{4}\cdot\frac{1}{\sqrt{2}}}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \frac{1}{\sqrt{2}}\left(1+\frac{\pi}{4}\right)\times 2 = \sqrt{2}\left(1+\frac{\pi}{4}\right).$$

**S12.** $y = \frac{x}{x+5} \Rightarrow y\cdot(x+5) = x$

$= \quad y\cdot\frac{d}{dx}(x+5) + (x+5)\cdot\frac{dy}{dx} = 1$

$= \quad y + (x+5)\cdot\frac{dy}{dx} = 1$

$= \quad (x+5)\cdot\frac{dy}{dx} = (1-y)$

$= \quad y\cdot(x+5)\cdot\frac{dy}{dx} = y(1-y)$

$= \quad x\cdot\frac{dy}{dx} = y(1-y).$

**Hence proved**.

**S13.** $y = \theta\cdot\sin\theta$

$\because$ Slope of tangent at point $x = x_0$ is given by $\left.\frac{dy}{dx}\right|_{x=x_0}$

$\therefore \quad y = \theta\cdot\sin\theta$

$\therefore \quad \frac{dy}{d\theta} = \left\{\frac{d}{d\theta}(\sin\theta)\right\}\cdot\theta + \sin\theta\cdot\frac{d\theta}{d\theta}$

$$= \theta \cdot \cos\theta + \sin\theta$$

$\therefore$ $$\left.\frac{dy}{d\theta}\right|_{\theta = \frac{\pi}{2}} = \frac{\pi}{2} \cdot \cos\frac{\pi}{2} + \sin\frac{\pi}{2} = 1.$$

**S14.** $\because$ $$\frac{d}{dx}(x^n) = nx^{n-1}$$

$\because$ $$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + 1$$

$$f'(x) = \frac{100x^{99}}{100} + \frac{99x^{98}}{99} + \ldots$$

$\therefore$ $$f'(x) = x^{99} + x^{98} + \ldots + x + 1$$

Now, $$f'(1) = 1 + 1 + \ldots \text{ to 100 term} = 100$$

$\therefore$ $$f'(0) = 1.$$

$\therefore$ $$f'(1) = 100 \cdot 1 = 100 \cdot f'(0)$$

$\therefore$ $$f'(1) = 100 \cdot f'(0).$$